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Interplay of vector-like top partner multiplets in a realistic mixing set-up

Giacomo Cacciapaglia,[a] **Aldo Deandrea,** [a,1] **Naveen Gaur,**[b] **Daisuke Harada,**[c] **Yasuhiro Okada,**[d,e] **Luca Panizzi**[f]

[a] *Université de Lyon, France; Université Lyon 1, CNRS/IN2P3, UMR5822 IPNL, F-69622 Villeurbanne Cedex, France.*

[b] *Department of Physics, Dyal Singh College (University of Delhi), Lodi Road, New Delhi - 110003, India*

[c] *Centre for High Energy Physics, Indian Institute of Science, Bangalore 560012, India*

[d] *KEK Theory Center, Institute of Particle and Nuclear Studies, KEK, 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

[e] *Department of Particle and Nuclear Physics, Graduate University for Advanced Studies (Sokendai), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

[f] *School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, UK*

E-mail: g.cacciapaglia@ipnl.in2p3.fr, deandrea@ipnl.in2p3.fr, gaur.nav@gmail.com, harada@cts.iisc.ernet.in, yasuhiro.okada@kek.jp, L.Panizzi@soton.ac.uk

Abstract: The ATLAS and CMS collaborations at the LHC have performed analyses on the existing data sets, studying the case of one vector-like fermion or multiplet coupling to the standard model Yukawa sector. In the near future, with more data available, these experimental collaborations will start to investigate more realistic cases. The presence of more than one extra vector-like multiplet is indeed a common situation in many extensions of the standard model. The interplay of these vector-like multiplet between precision electroweak bounds, flavour and collider phenomenology is a important question in view of establishing bounds or for the discovery of physics beyond the standard model. In this work we study the phenomenological consequences of the presence of two vector-like multiplets. We analyse the constraints on such scenarios from tree-level data and oblique corrections for the case of mixing to each of the SM generations. In the present work, we limit to scenarios with two top-like partners and no mixing in the down-sector.

[1] also Institut Universitaire de France, 103 boulevard Saint-Michel, 75005 Paris, France

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1 Introduction

The Large Hadron Collider (LHC) has confirmed the effective description of the electroweak sector given by the Standard Model (SM) Lagrangian with the discovery of the Higgs boson and the analysis of its properties. It also features a very strong potential for the discovery or exclusion of new physics/particles, thus opening the possibility of investigating both strongly and weakly coupled extensions of the Standard Model. New vector-like (VL) fermions are often present in many of the extensions of the SM, especially in relation with the top sector (top partners, as for example in composite Higgs models [1–4], extra-dimensional models [5–9], little Higgs models [10–12], gauge-Higgs models [13], gauge coupling unification [14, 15] and models with an extended custodial symmetry [16, 17]). Both CMS [18] and ATLAS [19] have recently devoted a considerable effort in the analyses apt to setting bounds on this type of new particles. Initially, simplifying assumptions were considered (mixing only with the third generation of SM quark family or specific decay modes) [20–29]. However the most recent analyses, due to larger data samples, allow exploring more general situations with mixing

of VL quarks with the first two generation of SM quarks [30–34]. Considering the presence of a complete multiplet of the symmetries of the Standard Model is however not enough in some realistic scenarios: in fact, theoretically justified models often contain a multiplet of larger global symmetries which can be described in terms of several multiplets which are close in mass. The various multiplets then mix with each other via the Higgs interactions. The presence of general mixing structures and the interplay of different multiplets typically affects the tree-level and loop-level bounds, thereby modifying the results expected by performing simplified analyses based on a single particle or a single multiplet. This work is devoted to the detailed exploration of general structures and mixing of more than one VL quark multiplet and, specifically, we study in detail the implications of the presence of *two* VL quark multiplets mixings with *any* of the 3 SM quark generation. We also focus on a specific sub-set of scenarios where both VL multiplets contain a top partner (*i.e.* with electric charge $+2/3\ e$), and where eventual bottom partners (i.e. with electric charge $-1/3\ e$) do not mix with the SM down sector. This choice is done to minimise the constraints from flavour, which are very severe on mixing in the down sector only. In the scenarios we selected, larger mixing angles are allowed thus providing larger single production cross sections at the LHC. These scenarios are also theoretically justified in models where the new physics couples dominantly to the top quark. Of course, depending upon the multiplet considered, non-SM quarks, *i.e.* quarks having non-SM electric charge, may be present in the considered multiplets. We have estimated the constraints on such scenarios from electroweak precision (EWP) data (oblique and non-oblique) and current LHC data.

The paper is organized as follows: in Section 2 we classify (through their $(SU(2)_L, U(1)_Y)$ quantum numbers) all the possible pairs of VL multiplets that can interact with SM quarks via a SM Higgs doublet. In Section 3 the Yukawa couplings of the VL multiplets with SM quark generations are described. In the same section we have also identified three kind of scenarios depending on the multiplet content, namely: top-type multiplets, bottom-type multiplets and mixed multiplets. The mass matrices of the cases we considered in our analysis are provided in Section 3.5. Section 4 is devoted to a discussion of the bounds and constraints on masses and mixing parameters we considered for the numerical analysis: in Section 4.1 we describe the tree-level bounds, while the bounds coming from oblique corrections, *i.e.* S and T parameters [35, 36], are discussed in Section 4.2. In Section 5 we present the results of our numerical analysis: in Sections 5.1-5.4 the bounds on parameter space from tree-level constraints and EWP tests are described, while in Section 5.5 we provide a comparison between the bounds from single VL top production at LHC14 with flavour and EWP bounds. We finally conclude in Section 6.

2 Vector-like multiplets

The minimal set of VL multiplets that can mix with SM quarks and a SM (or SM-like) Higgs boson have been extensively studied in literature [20, 24, 28, 30–32, 37]. In Tables 1-3 we list the $SU(2)_L \times U(1)_Y$ quantum numbers of the VL quark multiplets that can have interactions

Multiplet	ψ	$(SU(2)_L, U(1)_Y)$	T_3	Q_{EM}	Yukawa to SM
Singlet 2/3	U	$(\mathbf{1}, 2/3)$	0	$+2/3$	Yes
Doublet 7/6	$\begin{pmatrix} X^{5/3} \\ U \end{pmatrix}$	$(\mathbf{2}, 7/6)$	$+1/2$ $-1/2$	$+5/3$ $+2/3$	Yes
Triplet 5/3	$\begin{pmatrix} X^{8/3} \\ X^{5/3} \\ U \end{pmatrix}$	$(\mathbf{3}, 5/3)$	$+2$ $+1$ 0	$+8/3$ $+5/3$ $+2/3$	No

Table 1. Quantum numbers for the top–type VL multiplets (up to triplets), explicitly indicating weak isospin, hypercharge, electric charge (Q_{EM}) and if a direct Yukawa coupling to SM quarks is allowed.

– when taken alone or in pairs – with SM quark generations and Higgs boson doublet under $SU(2)_L \times U(1)_Y$ symmetry. The tables are organized as follows:

- top-type multiplets (Table 1): multiplets containing one VL top partner but no bottom partners (*i.e.* no VL quark with electric charge $-1/3\ e$). In addition to a top partner, these multiplets may contain quarks with exotic charges $5/3\ e$ and $8/3\ e$.
- bottom-type multiplets (Table 2): multiplets containing one VL bottom partner but no top partners (*i.e.* no VL quark with charge $2/3\ e$). In addition to a bottom partner these multiplets may contain quarks with exotic charges $-4/3\ e$ and $-7/3\ e$.
- mixed multiplets (Table 3): multiplets containing both VL top and bottom partners. In addition these multiplets may contain all of the exotic charged VL quarks.

The multiplets in these Tables constitute the building blocks we will use to construct scenarios with 2 VL multiplets [1].

Multiplet	ψ	$(SU(2)_L, U(1)_Y)$	T_3	Q_{EM}	Yukawa to SM
Singlet-1/3	D	$(\mathbf{1}, -1/3)$	0	-1/3	Yes
Doublet -5/6	$\begin{pmatrix} D \\ Y^{-4/3} \end{pmatrix}$	$(\mathbf{2}, -5/6)$	$+1/2$ $-1/2$	$-1/3$ $-4/3$	Yes
Triplet -4/3	$\begin{pmatrix} D \\ Y^{-4/3} \\ Y^{-7/3} \end{pmatrix}$	$(\mathbf{3}, -4/3)$	0 -1 -2	$-1/3$ $-4/3$ $-7/3$	No

Table 2. Quantum numbers for the bottom–type VL multiplets (up to triplets), explicitly indicating weak isospin, hypercharge, electric charge (Q_{EM}) and if a direct Yukawa coupling to SM quarks is allowed.

[1] A model where quarks and leptons were taken as a part of quadruplet is given in [38]

Multiplet	ψ	$(SU(2)_L, U(1)_Y)$	T_3	Q_{EM}	Yukawa to SM
Doublet 1/6	$\begin{pmatrix} U \\ D \end{pmatrix}$	$(\mathbf{2}, 1/6)$	$+1/2$ $-1/2$	$+2/3$ $-1/3$	Yes*
Triplet 2/3	$\begin{pmatrix} X^{5/3} \\ U \\ D \end{pmatrix}$	$(\mathbf{3}, 2/3)$	$+1$ 0 -1	$+5/3$ $+2/3$ $-1/3$	Yes
Triplet -1/3	$\begin{pmatrix} U \\ D \\ Y^{-4/3} \end{pmatrix}$	$(\mathbf{3}, -1/3)$	$+1$ 0 -1	$+2/3$ $-1/3$ $-4/3$	Yes
Quadruplet 7/6	$\begin{pmatrix} X^{8/3} \\ X^{5/3} \\ U \\ D \end{pmatrix}$	$(\mathbf{4}, 7/6)$	$+3/2$ $+1/2$ $-1/2$ $-3/2$	$+8/3$ $+5/3$ $+2/3$ $-1/3$	No
Quadruplet 1/6	$\begin{pmatrix} X^{5/3} \\ U \\ D \\ Y^{-4/3} \end{pmatrix}$	$(\mathbf{4}, 1/6)$	$+3/2$ $+1/2$ $-1/2$ $-3/2$	$+5/3$ $+2/3$ $-1/3$ $-4/3$	No
Quadruplet - 5/6	$\begin{pmatrix} U \\ D \\ Y^{-4/3} \\ Y^{-7/3} \end{pmatrix}$	$(\mathbf{4}, -5/6)$	$+3/2$ $+1/2$ $-1/2$ $-3/2$	$+2/3$ $-1/3$ $-4/3$ $-7/3$	No

Table 3. Quantum numbers for the mixed–type VL fermion multiplets (up to quadruplets), explicitly indicating weak isospin, hypercharge, electric charge (Q_{EM}) and if a direct Yukawa coupling to SM quarks is allowed. For the Doublet 1/6, one can write an independent Yukawa coupling with the right-handed up and down quarks.

3 New Yukawa couplings

The SM Yukawa couplings are the coefficients $y_u^{i,j}$ and $y_d^{i,j}$, where u and d refer to the coupling of the up-type and down-type quarks respectively and the indices i and j label the three SM generations ($i, j = 1, 2, 3$). These couplings allow the interactions of the SM quarks with the Higgs bosons according to the following Lagrangian terms:

$$\mathcal{L}_{SM} = -y_u^{i,j}\, \bar{Q}_L^i \tilde{H} u_R^j - y_d^{i,j}\, \bar{Q}_L^i H d_R^j + h.c.\,, \tag{3.1}$$

where $H = (\mathbf{2}, 1/2)$ is the Higgs boson doublet coupling to down-type quarks, $\tilde{H} = i\tau^2 H^*$ is the same Higgs multiplet coupling to up-type quarks, $Q_L = (\mathbf{2}, \frac{1}{6})$ is the SM quark doublet, $u_R = (\mathbf{1}, \frac{2}{3})$ and $d_R = (\mathbf{1}, -\frac{1}{3})$ are the SM singlets. After the Higgs boson gets its Vacuum Expectation Value (VEV) we obtain:

$$\mathcal{L}_{SM} = -\left(\tilde{m}^{up}\right)^{ij}\, \bar{u}_L^i u_R^j - \left(\tilde{m}^{down}\right)^{ij}\, \bar{d}_L^i d_R^j + h.c.\,; \tag{3.2}$$

where $\tilde{m}^{up}$ and $\tilde{m}^{down}$ are the SM up-type and down-type 3×3 mass matrices for quarks. In the following of the paper, we will work in the basis where the SM Yukawas are diagonal and the eigenvalues are real and positive. This implies that the phases of the SM quark fields are fixed (up to an overall phase – the Baryon number), and that a mixing matrix $\tilde{V}_{\rm CKM}$ appears in the couplings of the SM quark fields to the W boson. We anticipate that this mixing matrix is not the measured CKM matrix, because of the effect of mixing to the VL quarks.

The presence of VL multiplets allows us to add Yukawa interactions between the VL multiplets and the SM quarks. Due to $SU(2)$ products of representations, new quark doublets can couple with the SM right-handed singlets, while new quark singlets and triplets can couple to SM left-handed doublets. However, as we are considering a more general case in which more than one VL multiplet is present, new Yukawa interactions between two VL quark multiplets and the SM Higgs doublet appear.

In the following we will not considered two multiplets of same type (same hypercharge). As we are interested in the interplay of VL quarks, we have considered cases that satisfy the following conditions:

- there must be two VL top quarks.
- eventual VL bottom quarks do not mix with SM bottom sector, *i.e.* we can take the mixing to be zero without affecting the top sector, to ensure that the model is not constrained too much by the stringent flavour physics and Zbb coupling bounds from the bottom sector.

The latter conditions tells us that the only multiplet containing a bottom quark that we allow is the Doublet–1/6, for which the Yukawa involving the down sector is independent form the Yukawa involving the up sector. These conditions leave us with only four multiplet combinations, namely:

- Singlet ($Y = 2/3$) + Doublet ($Y = 7/6$);
- Doublet ($Y = 7/6$) + Triplet ($Y = 5/3$);
- Singlet ($Y = 2/3$) + Doublet ($Y = 1/6$);
- Doublet ($Y = 1/6$) + Doublet ($Y = 7/6$).

The analytical expressions of the mass matrices for the above combinations will be derived in the following. For the remaining cases with two different VL multiplets, the mass matrices are presented in Appendix A.

The notation we will use to refer to the new Yukawa and mass terms in the Lagrangian is the following:

- $\mathcal{L}_{V-SM}$: Yukawa interactions between a VL multiplet and SM quarks;
- $\mathcal{L}_{V-V}$: Yukawa interactions between two VL multiplets;
- $\mathcal{L}_{\rm mass}$: mass terms after the Higgs boson acquires its VEV and pure VL mass terms.

We will also denote the non-SM Yukawa couplings as:

- λ_I^k: Yukawa between the VL quark $I = 1, 2$ with the SM quark of generation k;
- λ_{Id}^k: Yukawa coupling of the Doublet–1/6 with the right handed bottom (this coupling will be assumed to be very small in our analysis);
- ξ_1: Yukawa between two VL quarks, involving a left-handed doublet (or quadruplet) and a right-handed singlet (or triplet);
- ξ_2: Yukawa between two VL quarks, involving a left-handed singlet (or triplet) and a right-handed doublet (or quadruplet).

After the Higgs develops its VEV, $\langle H\rangle = \frac{v}{\sqrt{2}}(0,1)^T$, these terms will generate mass terms mixing the VL quarks among themselves and with the SM quarks. In the mass matrices we will consistently use the following notation:

- $y_I^k = \lambda_I^k \frac{v}{\sqrt{2}}$, when the mixing involves a VL doublet (or quadruplet);
- $x_I^k = \lambda_I^k \frac{v}{\sqrt{2}}$, when the mixing involves a VL singlet (or triplet);
- $y_{Id}^k = \lambda_{Id}^k \frac{v}{\sqrt{2}}$, when the mixing involves a VL doublet (or quadruplet) of down type quark;
- $x_{Id}^k = \lambda_{Id}^k \frac{v}{\sqrt{2}}$, when the mixing involves a VL singlet (or triplet) of down type quark;
- $\omega = \xi_1 \frac{v}{\sqrt{2}}$ and $\omega' = \xi_2 \frac{v}{\sqrt{2}}$, for the mixing among VL multiplets.

Note that for VL multiplets with the same quantum numbers as the SM quarks, a direct mass mixing can be written down: however, this term is not physical, as it can be easily removed by redefining the fields corresponding to the SM and VL quarks. In the following, therefore, we will never consider this term in the Lagrangians. Finally, all the new Yukawa couplings are potentially complex couplings: for each case, we will specify the number of physical phases, recalling that we chose to work in the basis where the SM Yukwas are real, positive and diagonal (thus 3 mixing angles and one phase are already accounted for in $\tilde{V}_{\rm CKM}$). We also chose the VL masses to be real and positive, thus fixing the relative phase between the left and right-handed components of the VL quarks.

3.1 Singlet $Y = 2/3$ and Doublet $Y = 7/6$

The Doublet $Y = 7/6$ couples to Singlets $Y = 2/3$ (both SM and VL):

$$\mathcal{L}_{V-SM} = -\lambda_1^k \,\bar{\psi}_{1L} H u_R^k - \lambda_2^k \bar{Q}_L \tilde{H} \psi_{2R} + h.c.\,, \tag{3.3}$$
$$\mathcal{L}_{V-V} = -\xi_1 \,\bar{\psi}_{1L} H \psi_{2R} - \xi_2 \,\bar{\psi}_{1R} H \psi_{2L} + h.c.\,, \tag{3.4}$$

where $\psi_1 = (\mathbf{2}, \frac{7}{6}) = \begin{pmatrix} X_1^{5/3} & U_1 \end{pmatrix}^T$ and $\psi_2 = (\mathbf{1}, \frac{2}{3}) = U_2$. In this Lagrangian, we can use the relative phases between the two VL quarks to fix $\xi_1 > 0$, so that ξ_2 contains one physical phase. The relative phase between the VL and the SM quarks can be used to fix one of the 6 phases contained in $\lambda_{1,2}^k$. In total, the model has 6 additional phases to the SM ones, when all new Yukawas are non-vanishing. The mass Lagrangian is:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = & -y_1^k \bar{U}_{1L} u_R^k - x_2^k u_L^k U_{2R} - \omega \bar{U}_{1L} U_{2R} - \omega' \bar{U}_{1R} U_{2L} - M_1 \,\bar{U}_{1L} U_{1R} \\ & -M_2 \,\bar{U}_{2L} U_{2R} - M_1 \,\bar{X}_{1L}^{5/3} X_{1R}^{5/3} + h.c.\,.\end{aligned} \tag{3.5}$$

This leads to the mass matrix:

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & 0_{3\times1} & (x_2^k)_{3\times1} \\ (y_1^k)_{1\times3} & M_1 & \omega \\ 0_{1\times3} & \omega' & M_2 \end{pmatrix}, \quad M_{X_1^{5/3}} = M_1\,, \tag{3.6}$$

where $\tilde{m}^{up}$ is the SM 3×3 mass matrix of up sector. The mass matrix can be diagonalised by two unitarity matrices:

$$M_u = V_L^u \cdot M_u^{diag} \cdot (V_R^u)^\dagger\,. \tag{3.7}$$

The general procedure for the diagonalisation of mass matrices is described in Section 3.5.

3.2 Doublet $Y = 7/6$ and Triplet $Y = 5/3$

The Triplet $Y = 5/3$ couples to the Doublet $Y = 7/6$, which in turn couples to the SM singlet u_R:

$$\mathcal{L}_{V-SM} = -\lambda_1^k \,\bar{\psi}_{1L} H u_R^k + h.c.\,, \tag{3.8}$$
$$\mathcal{L}_{V-V} = -\xi_1 \,\bar{\psi}_{1L} \tau^a \tilde{H} (\psi_{2R})^a - \xi_2 \,\bar{\psi}_{1R} \tau^a \tilde{H} (\psi_{2L})^a + h.c.\,, \tag{3.9}$$

where $\psi_1 = (\mathbf{2}, \frac{7}{6}) = \begin{pmatrix} X_1^{5/3} & U_1 \end{pmatrix}^T$ and $\psi_2 = (\mathbf{3}, \frac{5}{3}) = \begin{pmatrix} X_2^{8/3} & X_2^{5/3} & U_2 \end{pmatrix}^T$. We can again use the relative phase between VL quarks to fix $\xi_1 > 0$ (and leave ξ_2 complex), and remove one of the 3 phases of λ_1^k, thus the model contains 3 additional physical phases. The mass contributions from the Yukawa interactions, including the VL masses, give the Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = & -y_1^k \bar{U}_{1L} u_R^k - \sqrt{2}\,\omega \bar{U}_{1L} U_{2R} - \omega \bar{X}_{1L}^{5/3} X_{2R}^{5/3} - \sqrt{2}\,\omega' \bar{U}_{1R} U_{2L} - \omega' \bar{X}_{1R}^{5/3} X_{2L}^{5/3} - M_1 \,\bar{U}_{1L} U_{1R} \\ & -M_1 \,\bar{X}_{1L}^{5/3} X_{1R}^{5/3} - M_2 \,\bar{U}_{2L} U_{2R} - M_2 \,\bar{X}_{2L}^{5/3} X_{2R}^{5/3} - M_2 \,\bar{X}_{2L}^{8/3} X_{2R}^{8/3} + h.c.\,,\end{aligned} \tag{3.10}$$

leading to the mass matrices:

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times 3} & 0_{3\times 1} & 0_{3\times 1} \\ (y_1^k)_{1\times 3} & M_1 & \sqrt{2}\,\omega \\ 0_{1\times 3} & \sqrt{2}\,\omega' & M_2 \end{pmatrix}, \quad M_{X^{5/3}} = \begin{pmatrix} M_1 & \omega \\ \omega' & M_2 \end{pmatrix}, \quad M_{X^{8/3}} = M_2\,. \tag{3.11}$$

Note that the mass matrix in the up sector is the same as the previous case (with $x_2 = 0$), while now there are two exotic charged quarks which mix via the Yukawa couplings $\xi_{1,2}$.

3.3 Singlet $Y = 2/3$ and Doublet $Y = 1/6$

In this case, the two VL multiplets have the same quantum numbers of the SM quarks, therefore one can replicate all the standard Yukawa couplings, including an independent coupling for the right-handed downs d_R:

$$\mathcal{L}_{V-SM} = -\lambda_1^k\,\bar{\psi}_{1L}\tilde{H}u_R^k - \lambda_{1d}^k\,\bar{\psi}_{1L}Hd_R^k - \lambda_2^k\bar{Q}_L^k\tilde{H}\psi_{2R} + h.c.\,, \tag{3.12}$$

$$\mathcal{L}_{V-V} = -\xi_1\,\bar{\psi}_{1L}\tilde{H}\psi_{2R} - \xi_2\,\bar{\psi}_{1R}\tilde{H}\psi_{2L} + h.c.\,, \tag{3.13}$$

where $\psi_1 = (\mathbf{2}, \frac{1}{6}) = (U_1\ D_1)^T$ and $\psi_2 = (\mathbf{1}, \frac{2}{3}) = U_2$. Like in the cases above, we can use the relative phases of the VL quarks to make $\xi_1 > 0$, and remove one of the 6 phases in $\lambda_{1,2}^k$. The bottom coupling λ_{1d}^k are also complex, however as explained above we will set these couplings to zero in the following. The mass terms take the form (where we normalise the mass parameters to have coefficient one for the top partners):

$$\begin{aligned} \mathcal{L}_{\text{mass}} = & -y_1^k\bar{U}_{1L}u_R^k - x_2^k u_L^k U_{2R} - \omega\bar{U}_{1L}U_{2R} - \omega'\bar{U}_{2L}U_{1R} \\ & -M_1\,\bar{U}_{1L}U_{1R} - M_2\,\bar{U}_{2L}U_{2R} - M_1\,\bar{D}_{1L}D_{1R} + h.c.\,. \end{aligned} \tag{3.14}$$

The mass matrices, therefore, read:

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times 3} & 0_{3\times 1} & (x_2^k)_{3\times 1} \\ (y_1^k)_{1\times 3} & M_1 & \omega \\ 0_{1\times 3} & \omega' & M_2 \end{pmatrix}, \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times 3} & 0_{1\times 3} \\ (0)_{3\times 1} & M_1 \end{pmatrix}. \tag{3.15}$$

Now, the mass matrix in the up sector is the same as the first case, while the down sector mass matrix is diagonal (as we set to zero the mixing in the down sector). No exotic charges are present in this case.

3.4 Doublet $Y = 1/6$ and Doublet $Y = 7/6$

This is the only case where we consider two doublets, thus both VL multiplets only couple to the right-handed SM quarks:

$$\mathcal{L}_{V-SM} = -\lambda_1^k\,\bar{\psi}_{1L}\tilde{H}u_R^k - \lambda_{1d}^k\,\bar{\psi}_{1L}Hd_R^k - \lambda_2^k\,\bar{\psi}_{2L}Hu_R^k + h.c.\,, \tag{3.16}$$

where $\psi_1 = (\mathbf{2}, \frac{1}{6}) = (U_1\ D_1)^T$ and $\psi_2 = (\mathbf{2}, \frac{7}{6}) = \left(X_2^{5/3}\ U_2\right)^T$. In this case, no Yukawa coupling between the two VL multiplet is allowed, therefore one can use the two free phases

to remove one phase in λ_1^k and one in λ_2^k, so that only 4 new phases are present in this model. Once again, we will set $\lambda_{1d}^k = 0$. The mass Lagrangian and mass matrices become:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = & -y_{1u}^k \bar{U}_{1L} u_R^k - y_{1d}^k \bar{D}_{1L} d_R^k - y_2^k U_{2L} u_R^k \\ & -M_1\, \bar{U}_{1L} U_{1R} - M_1\, \bar{D}_{1L} D_{1R} - M_2\, \bar{U}_{2L} U_{2R} - M_2\, \bar{X}_{2L}^{5/3} X_{2R}^{5/3} + h.c.\,,\end{aligned} \tag{3.17}$$

and

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times 3} & 0_{3\times 1} & 0_{3\times 1} \\ (y_1^k)_{1\times 3} & M_1 & 0 \\ (y_2^k)_{1\times 3} & 0 & M_2 \end{pmatrix}, \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times 3} & 0_{3\times 1} \\ (0)_{1\times 3} & M_2 \end{pmatrix}, \quad M_{X^{5/3}} = M_2\,. \tag{3.18}$$

The structure of the up-sector mass matrix is now different from the cases above.

3.5 Diagonalisation of the mass matrices

We have discussed so far four special cases, where two top partners mix with the up-sector giving rise to 5×5 mass matrices, while the down sector is always diagonal. More general cases, and the form of the mixing matrices, can be found in Appendix A. In the up sector, the mass matrices can be diagonalised by two unitary 5×5 matrices:

$$M_u = V_L \cdot M_u^{diag} \cdot V_R^\dagger\,, \tag{3.19}$$

with:

$$M_u^{diag} = \begin{pmatrix} m_u & & & & \\ & m_c & & & \\ & & m_t & & \\ & & & m_{t'_1} & \\ & & & & m_{t'_2} \end{pmatrix}. \tag{3.20}$$

Indeed when two VL multiplet are present at the same time, there are three types of mixing structures which can arise with the SM up quarks:

- Case A: two semi-integer isospin multiplets (as doublets, quadruplets, etc.). In this case the mass matrix becomes:

$$M_u^{(A)} = \begin{pmatrix} \tilde{m}_u & & & 0 & 0 \\ & \tilde{m}_c & & 0 & 0 \\ & & \tilde{m}_t & 0 & 0 \\ y_1^1 & y_1^2 & y_1^3 & M_1 & 0 \\ y_2^1 & y_2^2 & y_2^3 & 0 & M_2 \end{pmatrix}; \tag{3.21}$$

- Case B: two integer isospin multiplets (as singlets, triplets, etc.). The mass matrix is:

$$M_u^{(B)} = \begin{pmatrix} \tilde{m}_u & & & x_1^1 & x_2^1 \\ & \tilde{m}_c & & x_1^2 & x_2^2 \\ & & \tilde{m}_t & x_1^3 & x_2^3 \\ 0 & 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & 0 & M_2 \end{pmatrix}; \tag{3.22}$$

- Case C: one semi-integer isospin multiplet and one integer isospin multiplet. The mass matrix is:

$$M_u^{(C)} = \begin{pmatrix} \tilde{m}_u & & & 0 & x_2^1 \\ & \tilde{m}_c & & 0 & x_2^2 \\ & & \tilde{m}_t & 0 & x_2^3 \\ y_1^1 & y_1^2 & y_1^3 & M_1 & \omega \\ 0 & 0 & 0 & \omega' & M_2 \end{pmatrix} . \tag{3.23}$$

The VL multiplets considered in our analysis belong to the cases indicated in Table 4, where the combinations we are considering for our numerical analysis have been highlighted. These mass matrices cannot be diagonalised analytically. One can obtain approximate results in the limit where the VL masses $M_{1,2}$ are much larger than the contribution from the Yukawa couplings, and general results can be found in the Appendix of Ref. [32]. In our numerical results, however, we will use a numerical procedure to find the correct mass eigenstates and mixing angles, detailed in the next subsection.

2nd ↓ 1st →	Singlet $Y = \frac{2}{3}$	Doublet $Y = \frac{1}{6}$	Doublet $Y = \frac{7}{6}$	Triplet $Y = \frac{5}{3}$
Singlet $Y = \frac{2}{3}$	case B	**case C**	**case C**	case B
Doublet $Y = \frac{1}{6}$		case A	**case A**	case C
Doublet $Y = \frac{7}{6}$			case A	**case C**
Triplet $Y = \frac{5}{3}$				case B

Table 4. Mixing structures for the VL quarks multiplets considered in our analysis. The combinations we have studied in detail are in bold.

Some models also contain two exotic quarks which mix via a 2×2 matrix, like in Section 3.2:

$$M_{X^{5/3}} = \begin{pmatrix} M_1 & \omega \\ \omega' & M_2 \end{pmatrix} . \tag{3.24}$$

This mass matrix can be diagonalised by two Unitary matrices

$$M_X = V_{XL} \cdot M_X^{diag} \cdot V_{XR}^\dagger , \tag{3.25}$$

with eigenvalues

$$M_{X_{1,2}}^2 = \frac{1}{2}\left(M_1^2 + M_2^2 + \omega^2 + |\omega'|^2 \mp \sqrt{(M_1^2 + M_2^2 + \omega^2 + |\omega'|^2)^2 - 4|M_1 M_2 - \omega\omega'|^2} \right) . \tag{3.26}$$

Parametrising the mixing matrices as

$$V_{XL/R} = \begin{pmatrix} \cos\alpha_{L/R} & e^{i\delta_{L/R}} \sin\alpha_{L/R} \\ -e^{-i\delta_{L/R}} \sin\alpha_{L/R} & \cos\alpha_{L/R} \end{pmatrix} ; \tag{3.27}$$

the mixing angles and phases can be expressed as

$$e^{-i\delta_L}\sin(2\alpha_L) = 2\frac{M_1\omega' + M_2\omega}{M_{X_2}^2 - M_{X_1}^2}\,, \quad e^{-i\delta_R}\sin(2\alpha_R) = 2\frac{M_1\omega + M_2\omega'}{M_{X_2}^2 - M_{X_1}^2}\,. \tag{3.28}$$

3.5.1 Numerical procedure

In order to evaluate the constraints and estimate the production cross-sections of VL quarks at colliders we need to write the Lagrangians presented in Section 3.1–3.4 in the mass basis. In the SM the physical masses of quarks are uniquely defined by Yukawa couplings. The introduction of VL quarks with couplings to all the three SM quark generations enlarges the mass matrices and results in variation of the physical masses of SM quarks. In Section 3.5 we have presented the structure of the 5×5 mass matrices in the gauge basis. The procedure we have adopted for diagonalisation is the following:

- **Step 1:** using the new Yukawa Couplings $(x_i, y_i, \omega, \omega')$ and the masses M_1, M_2 as input parameters we write the five eigenvalue equations (one for each of the physical quarks):

$$|M_u^\dagger M_u - \lambda_i I| = 0\,, \tag{3.29}$$

  where M_u are the mass matrices and λ_i are the eigenvalues (square of physical masses).

- **Step 2:** the first three equations correspond to the three known values of SM quark physical masses (m_u, m_c, m_t). We solve these equations for the three SM Yukawa couplings, namely $\tilde{m}_u, \tilde{m}_c, \tilde{m}_t$.

- **Step 3:** we insert the SM Yukawa couplings obtained in Step 2 into the mass matrices and diagonalize them again to get the mixing matrices (V_L^u and V_R^u) and the physical masses of the VL quarks.

In this process we only consider positive solution for the SM Yukawa couplings (Step 2), to be consistent with our original choice. For simplicity, we also set all the new physical phases to zero, allowing ourselves only a negative sign of the Yukawa couplings when physically inequivalent to the positive sign.

4 Tree-level and Electroweak Precision bounds

4.1 Tree-level bounds on VL quarks

In order to study the tree-level bounds we need to recall the couplings of the VL fermions to the gauge bosons and in particular to the Z boson. The complete structure is given in the appendix B. The mass matrices are diagonalised as follows:

$$M_u = V_L \cdot M_u^{diag} \cdot V_R^\dagger\,, \tag{4.1}$$

so that the mass eigenstates are defined as:

$$\begin{pmatrix} u \\ c \\ t \\ t'_1 \\ t'_2 \end{pmatrix}_{L/R} = V^{\dagger}_{L/R} \cdot \begin{pmatrix} u^1 \\ u^2 \\ u^3 \\ U_1 \\ U_2 \end{pmatrix}_{L/R} . \tag{4.2}$$

In the mass eigenstate basis, the couplings of the Z boson read (for the up-quarks):

$$g^{IJ}_{ZL} = \frac{g}{2\cos\theta_W}\left[\left(1 - \frac{4}{3}\sin^2\theta_W\right)\delta^{IJ} + \sum_{K=4,5}(2T_3^{(K)} - 1)V_L^{*,KI}V_L^{KJ}\right] , \tag{4.3}$$

$$g^{IJ}_{ZR} = \frac{g}{2\cos\theta_W}\left[\left(- \frac{4}{3}\sin^2\theta_W\right)\delta^{IJ} + \sum_{K=4,5}2T_3^{(K)}V_R^{*,KI}V_R^{KJ}\right] , \tag{4.4}$$

where $T_3^{(K)}$ is the weak isospin of the VL quark K. Note that the modifications to the couplings with respect to the SM values (including off-diagonal terms) are all proportional to the $V^{4I}_{L/R}$ and the $V^{5I}_{L/R}$ elements of the mixing matrices. Let's now consider the bounds applied generation by generation.

4.1.1 Bounds on the first generation

Atomic Parity Violation

The weak charge of a nucleus can be, in general, written as [39]:

$$Q_W = (2Z + N)(\tilde{g}^u_{ZL} + \tilde{g}^u_{ZR}) + (Z + 2N)(\tilde{g}^d_{ZL} + \tilde{g}^d_{ZR}) , \tag{4.5}$$

where $g_Z = \frac{g}{2\cos\theta_W}\tilde{g}_Z$. In our case:

$$\delta Q_W = (2Z + N)\sum_{K=4,5}\left((2T_3^{(K)} - 1)|V_L^{K1}|^2 + 2T_3^{(K)}|V_R^{K1}|^2\right) . \tag{4.6}$$

The strongest bound is for Cesium, for which $2Z + N = 188$, and [40]:

$$Q_W|_{\text{exp.}} = -73.20 \pm 0.35 , \qquad Q_W|_{SM} = -73.15 \pm 0.02 . \tag{4.7}$$

Neglecting the theoretical error on the SM value, which is rather small:

$$\delta Q_W = -0.05 \pm 0.35 . \tag{4.8}$$

4.1.2 Bounds on the second generation

Z-couplings measured at LEP

The couplings of the charm have been well measured at LEP [41]:

$$g^c_{ZL} = 0.3453 \pm 0.0036\,, \quad g^c_{ZR} = -0.1580 \pm 0.0051\,, \quad \text{correlation} = 0.30\,. \tag{4.9}$$

One can use this input to reconstruct a χ^2 distribution, and set bounds on the mixing angles: the most conservative approach is to assume that the central values correspond to the SM prediction, therefore any deviation must be smaller than the quoted errors. The χ^2 can be constructed as follows:

$$\chi^2 = \sum_{i,j=1,2} \delta g^i (V^{-1})^{ij} \delta g^j\,, \tag{4.10}$$

where δg are the deviations in the two couplings (left and right handed) and:

$$V^{ij} = \rho^{ij} \sigma^i \sigma^j\,, \quad \text{where} \quad \rho = \begin{pmatrix} 1 & 0.30 \\ 0.30 & 1 \end{pmatrix}\,, \tag{4.11}$$

and σ^j are the errors. In the plots below, we will draw confidence levels at 68% ($\chi^2 = 2.30$ for 3 degrees of freedom), 95% (5.99) and 99% (9.21).

4.1.3 Bounds on the third generation

W_{tb} couplings measured at TeVatron and LHC

As there is no mixing in the bottom sector, the value of V_{tb} is affected only by the mixing of the top with the VL quarks in the left-handed sector:

$$|V_{tb}|^2 = 1 - \sum_{K=4,5} |V_L^{K3}|^2\,. \tag{4.12}$$

A list of up-to-date direct measurements and lower bounds on V_{tb} can be found here [42]. The strongest bound is from a CMS measurements of single top cross sections at 7 TeV [43], which gives $|V_{tb}| > 0.92$ at 95% CL. We will use this limit to define the allowed region, even though all other searches, including Tevatron [44], CMS [45] and ATLAS [46] at 8 TeV, have bounds $|V_{tb}| > 0.80$ at 95% CL (see also [47, 48] for summaries of the most recent results).

4.2 Oblique corrections

In the following we analyse the impact of the interplay of two VL multiplets with the complete three SM generations in order to establish bounds from the electroweak precision tests (EPW) in term of the oblique corrections. These bounds will be compared with those coming from tree-level observables. In order to parameterise the effect of the loop correction we will use the Peskin-Takeuchi S, T and U parameters, defined as [35, 36]:

$$S = 16\pi \left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right]\,, \tag{4.13}$$

$$T = \frac{4\pi}{s_W^2 c_W^2 m_Z^2} \left[\Pi_{11}(0) - \Pi_{33}(0)\right]\,, \tag{4.14}$$

$$U = 16\pi \left[\Pi'_{11}(0) - \Pi'_{33}(0)\right]\,. \tag{4.15}$$

where Π_{ij} are the scalar two point functions, related to the W, Z, and A one–loop two point functions by:

$$\Pi_{AA} = e^2\Pi_{QQ}\,, \tag{4.16}$$
$$\Pi_{ZA} = \frac{e^2}{s_W c_W}\left(\Pi_{3Q} - s_W^2\Pi_{QQ}\right)\,, \tag{4.17}$$
$$\Pi_{ZZ} = \frac{e^2}{s_W^2 c_W^2}\left(\Pi_{33} - 2s_W^2\Pi_{3Q} + s_W^4\Pi_{QQ}\right)\,, \tag{4.18}$$
$$\Pi_{WW} = \frac{e^2}{s_W^2}\Pi_{11}\,. \tag{4.19}$$

For this work we have taken the SM reference point masses to be $m_{h,\text{ref}} = 126$ GeV, $m_{t,\text{ref}} = 173$ GeV and $m_{b,\text{ref}} = 4.2$ GeV. If $\hat{S}_{VL}$ and $\hat{T}_{VL}$ is the contribution of the model (including VL quarks) to the S and T parameters then the deviations can be defined as [49]:

$$S = \hat{S}_{VL} - \hat{S}_{SM}\,, \quad T = \hat{T}_{VL} - \hat{T}_{SM}\,, \tag{4.20}$$

where the SM reference values, $\hat{S}_{SM}$ and $\hat{T}_{SM}$, can be approximated as:

$$\hat{S}_{SM} \simeq \frac{N_c}{6\pi}\left[3 - \frac{1}{3}\ln\left(\frac{m_t^2}{m_b^2}\right)\right]\,, \tag{4.21}$$
$$\hat{T}_{SM} \simeq \frac{N_c}{16\pi s_W^2 c_W^2 m_Z^2}\left[m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\left(\frac{m_t^2}{m_b^2}\right)\right]\,. \tag{4.22}$$

By fixing $U = 0$, the experimental results for the S and T parameters are given by [50, 51]:

$$S = 0.05 \pm 0.09\,, \qquad T = 0.08 \pm 0.07\,, \tag{4.23}$$

where the correlation between S and T in this fit is $\rho_{ST} = 0.91$.

If VL multiplets are present in the physical spectrum, the Π_{ij} two-point functions get extra contributions from the new particles circulating in the loops. Obviously if other particles than the VL quarks are present in a specific model of new physics, they may also contribute. Therefore the bounds obtained from the S and T parameters should be taken with this restriction in mind. The detailed formulas for the contributions of the VL particles to the S and T parameters are given in Appendix C. In the numerical study we have combined the tree-level and loop-level bounds from the S and T parameters, in the case of two multiplets of VL quarks. This allows to study the effect of the interplay of the two multiplets and the effect of the extra Yukawa couplings among the two VL multiplets (ω and ω' parameters). Even if in the previous sections we have calculated analytically the general mixing structure under some approximations, for the numerical part of this analysis the mixing angles have been computed exactly by numerically diagonalising the mass matrices.

5 Results

For our numerical analysis we have considered the mass parameters M_1 and M_2 of the VL quarks to be same, *i.e.* $M_1 = M_2 = M$. As the models have too many parameters to make a meaningful scan, we will show results in two limiting cases, when possible:

- the VL quarks can mix with a single SM generation, but not with each other (i.e. $\omega \sim \omega' \ll M$);
- the VL quarks mix with each other (wherever possible), but the mixing with SM quarks is very small.

The results in these simple limits can give a general idea on the allowed value of the mixing parameters, even though the case where all of them are non-zero is more realistic. We will focus on the benchmark value for the VL mass of 800 GeV as a recent CMS analysis [52] sets a bound of 788 GeV under the assumption of strong pair production of VL quarks and 100% branching fractions to qW. In the following we also consider cases in which the VL quark does not decay to qW. For these cases the bound does not apply directly, but these VL quarks are in doublets containing also other VL quarks for which the bound applies. As it is reasonable to assume that mass splittings inside multiplets are not large compared to the mass scale of the multiplet, we shall apply this 800 GeV benchmark value to all cases.

5.1 Singlet $Y = 2/3$ and Doublet $Y = 7/6$

This scenario contains – besides the SM particle spectrum – two VL top quarks and one exotic quark with charge 5/3. The Yukawa couplings and mass matrices for this scenario are given in Section 3.1. The additional parameters (apart from the SM ones) are: x_2^k, y_1^k, ω, ω' and M with k running on SM quark generations. We first study the case where $\omega' \sim \omega \sim 0$, and the VL quarks couple to a single generation: in this case, setting ω' to zero allows us to set both Yukawa couplings x_2 and y_1 to be real and positive. The allowed regions in the parameter space, given the constraints from tree-level and EWP tests discussed in Section 4, are presented in Figure 1. We see that for couplings to the light generations (see Appendix D for the plot with second generation), the tree level bounds always dominate, and require the mixing of VL quarks to be rather small. The case of the third generation is very different: the tree level bounds are very weak as they only come from V_{tb}, while EWP tests allow for large mixings, especially via a compensation between the doublet and singlet (in particular, y_1^3 can assume very large values). This situation can be very interesting in the single-production channel, where for instance the top partner may be produced via couplings to the first generation (the smaller coupling is easily compensated by the valence quark in the initial state [32]) and then decay into a third generation quark [53–55].

In Figure 2 we show the EWP bounds in the plane of the Yukawa couplings between VL quarks, ω and ω', assuming that the other couplings are small. This plot gives a general idea on the allowed size of ω and ω': the bound is indeed not very strong, and values up to

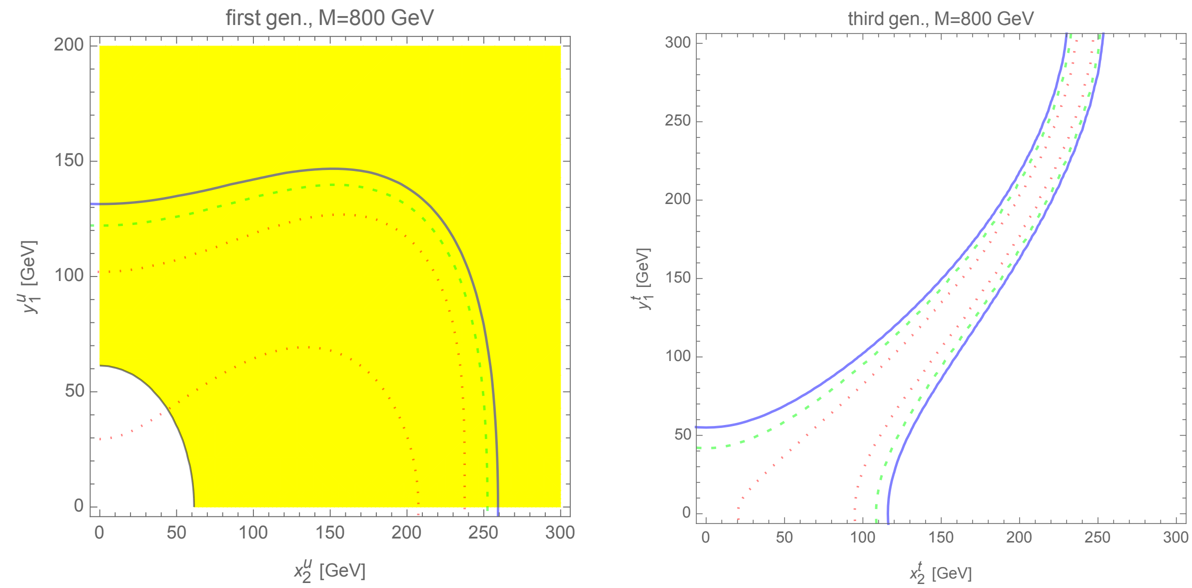

Figure 1. **Singlet $\boldsymbol{Y = 2/3}$ and Doublet $\boldsymbol{Y = 7/6}$** (Section 3.1): EWP bounds at 1 σ (red-dashed), 2 σ (green-dashed) and 3 σ (blue) for VL quarks coupling with the first (left panel) and third (right panel) SM generations, compared with the region excluded at 3σ by tree-level bounds (yellow region). Here, $M = 800$ GeV, and $\omega = \omega' = 0$. Only the first quadrant is shown as the figures are symmetric with respect to a sign change in the coordinates in the other 3 quadrants. Similar considerations apply to all the other figures of the same type.

300 GeV are allowed. The plot is clearly symmetric under change of sign of either ω or ω', reflecting the one arbitrary phase in this sector. As we are approximately decoupling the two t's from the SM quarks, the mixing is dominated by the 2×2 block of the VL quarks, similar to the matrix in Eq. (3.24). We then see that the mixing angles (Eq.(3.28)) vanish when $\omega = -\omega'$, thus explaining the sharp dents in the excluded region. This effect only appears in our limiting choice $M_1 = M_2$ and for negligible mixing to SM quarks.

5.2 Doublet $Y = 7/6$ and Triplet $Y = 5/3$

This scenario contains two VL top quarks and three exotic quarks: two with charge 5/3 and one with charge 8/3. All of these states contribute to the corrections to the EWP tests. The Yukawa couplings and mass matrices for this scenario are given in Section 3.2. The additional Yukawa couplings in the model are: y_1^k, ω, ω' and M with k running on SM quark generations.

As there is a single Yukawa mixing involving the SM quarks, we decided to add a non-vanishing ω in the scan, setting $\omega' = \omega$. The combined bounds from tree-level and EWP tests are given in Figure 3 for scenarios where the VL quarks mix with either one of the SM quark generation (case for second generation in Appendix D). While three level bounds only exclude a large mixing to the SM quarks in the case of light generations, a bound on the VL Yukawa ω arises from the EWP bounds. For the light generations, both Yukawas are constrained to

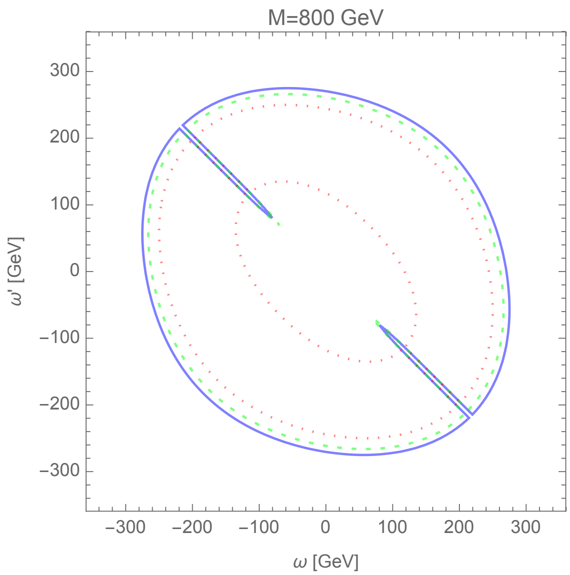

Figure 2. **Singlet $\boldsymbol{Y = 2/3}$ and Doublet $\boldsymbol{Y = 7/6}$** (Section 3.1): EWP bounds at 1σ (red-dashed), 2σ (green-dashed) and 3 σ (blue) as a function of the new Yukawa couplings ω and ω' with $M = 800$ GeV. We have assumed that there is no mixing of VL quarks with the SM quark generations *i.e.* $x_2^k = y_1^k = 0$.

be small. For third generation, the EWP bounds give similar value near the axes, however there is a cancellation absent in the case of light generation which opens the parameter space for $\omega = \omega' \sim y_1^3$, so that larger mixing angles are allowed.

In Figure 4 we show the EWP bounds in the plane of the new Yukawa couplings ω and ω'. The bounds are not very strong, allowing values up to 200 GeV. We also observe, like in Figure 2, two dents for $\omega = -\omega'$ due to the vanishing of the mixing angles (for $M_1 = M_2$). This is again an artifact of our choice of equal VL masses and vanishing couplings to the SM quarks.

5.3 Singlet $Y = 2/3$ and Doublet $Y = 1/6$

This scenario contains a VL copy of the SM quarks: two VL top quarks and one VL bottom quark. The additional Yukawa couplings in the model, described in Section 3.3, are: y_1^k, x_2^k, and M with k running on SM quark generations. Here we set the Yukawa in the down sector $y_{1d}^k = 0$ to minimise bounds from flavour physics: this can be done independently on the up sector.

In Figure 5 we show the combined bounds from EWP tests for VL quarks mixing with individual SM quark generations. As in the previous scenarios, the mixing between VL quarks is taken to be zero, *i.e.* $\omega = \omega' = 0$. This scenario exhibits quite distinctive features depending on the VL mixings and masses. In the case of mixing with the third generation, EWP bounds constrain quite tightly the allowed mixing parameters. In the case of mixing

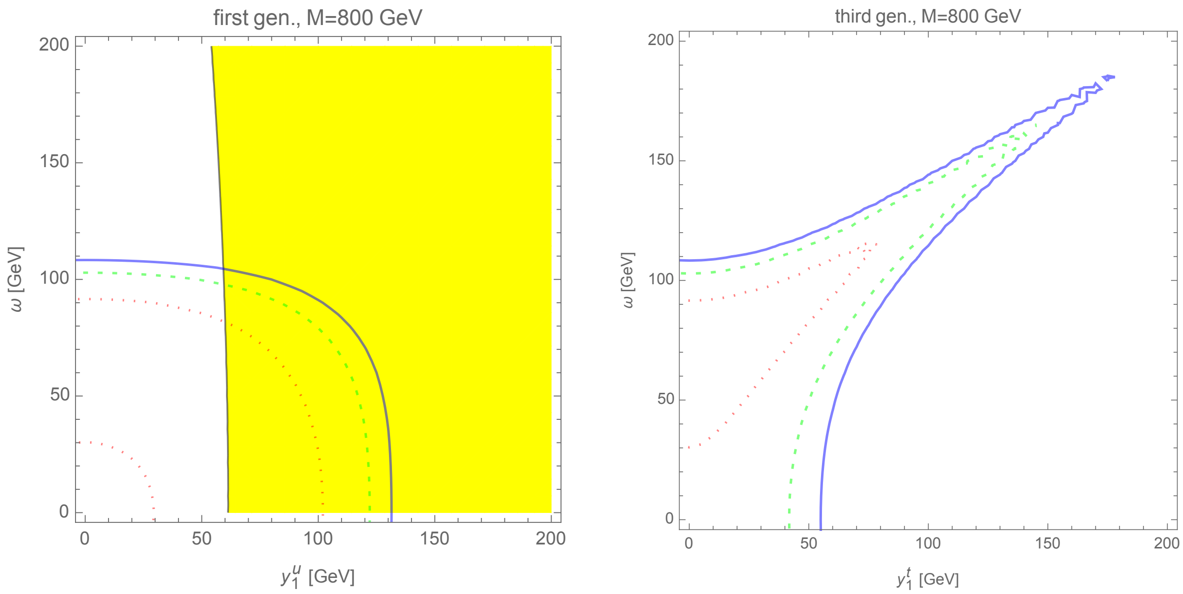

Figure 3. **Doublet $Y = 7/6$ and Triplet $Y = 5/3$** (Section 3.2): EWP bounds at 1σ (red-dashed), 2σ (green-dashed) and 3 σ (blue) for VL quarks coupling with the first (left panel) and third (right panel) SM generations, compared with the region excluded at 3σ by tree-level bounds (yellow region in the left panel). $M = 800$ GeV and $\omega = \omega'$.

with first generation only, tree level bounds are quite tight too up to a cancellation for $y_1^1 \sim x_2^1$ where the mixing can be arbitrarily large. This throat may suggest the possibility of large compositeness in the light quark sector. However, we see that EWP bounds exclude this region and point back to small mixing. For the case of mixing with second generation (see Appendix D), EWP bounds dominate and again force the scenario to small mixing parameters.

The bounds on ω and ω' are very similar to the previous cases, and we do not show them here.

5.4 Doublet $Y = 1/6$ and Doublet $Y = 7/6$

This scenario is particularly interesting as it corresponds to a bi-doublet of the custodial SO(4) symmetry, which is often a basic ingredient for top partial compositeness in models of composite Higgs (see for instance [26]). This scenario contains two VL top quarks, one VL bottom quark and one exotic quark with charge 5/3. The additional parameters, described in Section 3.4, are: y_1^k, y_2^k and M with k running on SM quark generations. Note that for this scenario mixing between VL quarks (*i.e.* ω and ω') is not allowed. Analogously to the model discussed in the previous Section 5.3, this model also introduces an additional mixing in the bottom sector, which is again independent from the the mixing in the top sector. Hence it is possible to impose the condition $y_{1d}^k = 0$ without affecting the top sector.

The results for the combined tree-level and EWP bounds are given in Figure 6. For the

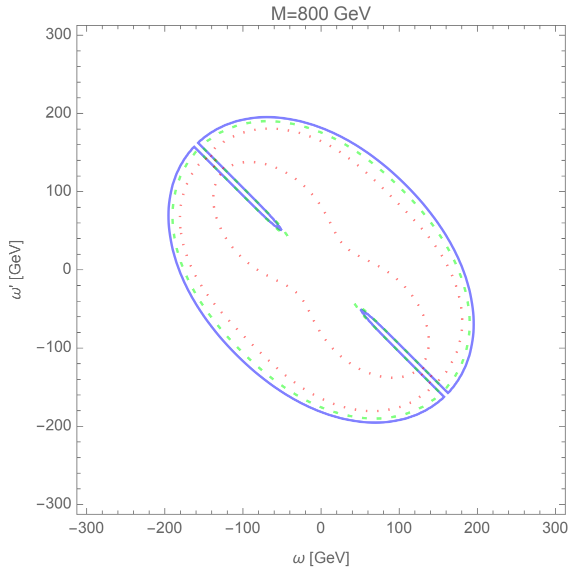

Figure 4. **Doublet $Y = 7/6$ and Triplet $Y = 5/3$** (Section 3.2): EWP bounds at 1σ (red-dashed), 2σ (green-dashed) and 3 σ (blue) as a function of the new Yukawa couplings ω and ω' with $M = 500/800$ GeV (left/right panel respectively). In addition the mixing of VL quarks with SM quarks is taken to be zero *i.e.* $y_1^k = 0$.

first generation (and the second, see Appendix D), there is an interesting cancellation in the tree-level bounds for $|y_1^k| = |y_2^k|$: this is a consequence of an enhanced custodial symmetry, and this fact has been used in the literature to justify $\mathcal{O}(1)$ mixings of VL quarks with light generations [56]. EWP bounds show a similar cancellation, however along an axes which is a bit off compared to $|y_1^k| = |y_2^k|$, therefore a tension between the two allowed regions develops for large mixings. Similar behaviour in the EWP bounds can be seen in the case of mixing to the third generation only.

5.5 Single production cross sections

In this section a comparison between the tree-level and loop-level bounds and the bounds from single production processes at the LHC is provided for the scenarios above. The relevance of single production is given by the fact that its cross-section depends on both the masses of the VL quarks and their couplings to the SM quarks; moreover, it is well known that single production becomes the dominant channel at the LHC, overcoming QCD pair production, when quark masses are higher than a certain (model-dependent) value. For typical scenarios where VL quarks mix predominantly with third generation and mixing parameters are not too constrained by flavour physics and EWP tests, the mass bounds from QCD pair production are already in the region where the single production channel is relevant or even dominant [30]. So far, few experimental searches for single production of VL quarks have been performed. The ATLAS experiment has performed two searches including single production of VL quarks: in

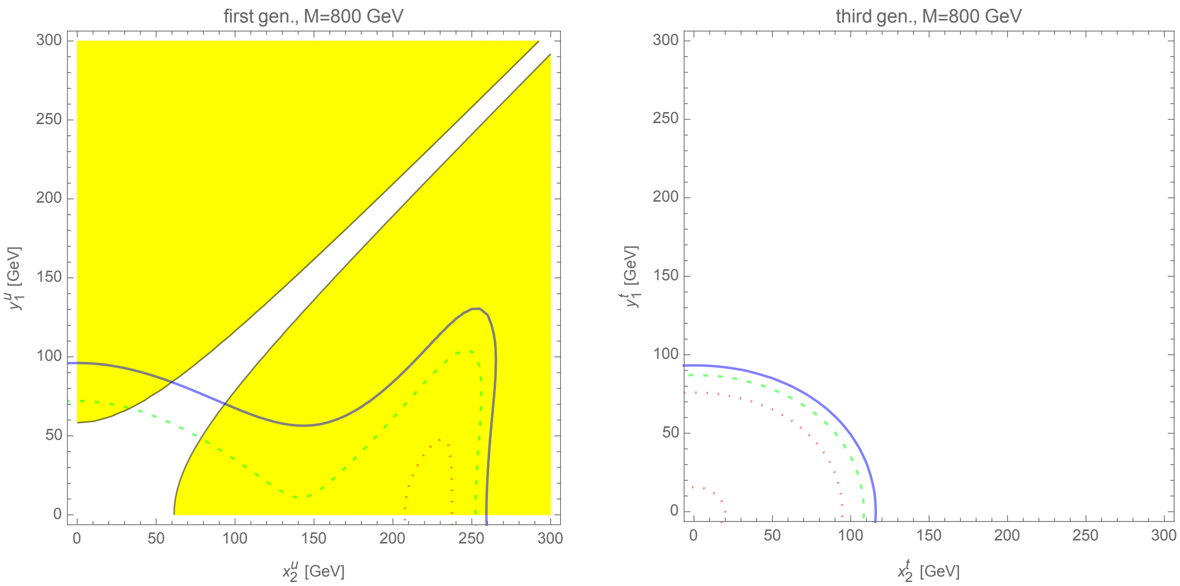

Figure 5. **Singlet $Y = 2/3$ and Doublet $Y = 1/6$** (Section 3.3): EWP bounds at 1σ (red-dashed), 2σ (green-dashed) and 3 σ (blue) for VL quarks coupling with the first (left panel) and third (right panel) SM generations, compared with the region excluded at 3σ by tree-level bounds (yellow region in the left panel). Here, $M = 800$ GeV, and $\omega = \omega' = 0$.

[57] a search for singly-produced VL quarks coupling only with first generation is performed, while [58] is a search for for pair+single production of VL quarks mixing with third generation only. The search [57] has already been considered in a previous analysis [32] for comparing bounds from LHC and flavour physics, and we will consider in this analysis part of the results obtained in that study. To be specific, in the following we will consider the single production of a VL top partner in association with a light jet, and the mass of the VL quark will be fixed to 800 GeV. We consider exclusive coupling to each of the three SM generations for each scenario.

The LHC bounds have been obtained by applying the model-independent parametrisation described in [32]. Considering the observed cross-section reported in the ATLAS analysis [57] and the universal coefficients computed in [32] (Tables 12-17) it is possible to set bounds on the overall coupling strengths of the singly-produced VL quarks by using the relations in Sections 3 and 4 of [32].

In the plots presented in Figs 7, 8, 9, 10 the LHC bounds are directly compared with the tree-level and EWPT bounds. The region inside the red line is the one allowed by the S and T parameters (oblique corrections) whereas the blue line marks the constraint from tree-level bounds. The EWPT bound should be taken only as an indication are the explicit assumption that no other extra states contribute to the corrections is imposed; this simplification is not true in general, e.g. in a complete model containing other new particles besides VL

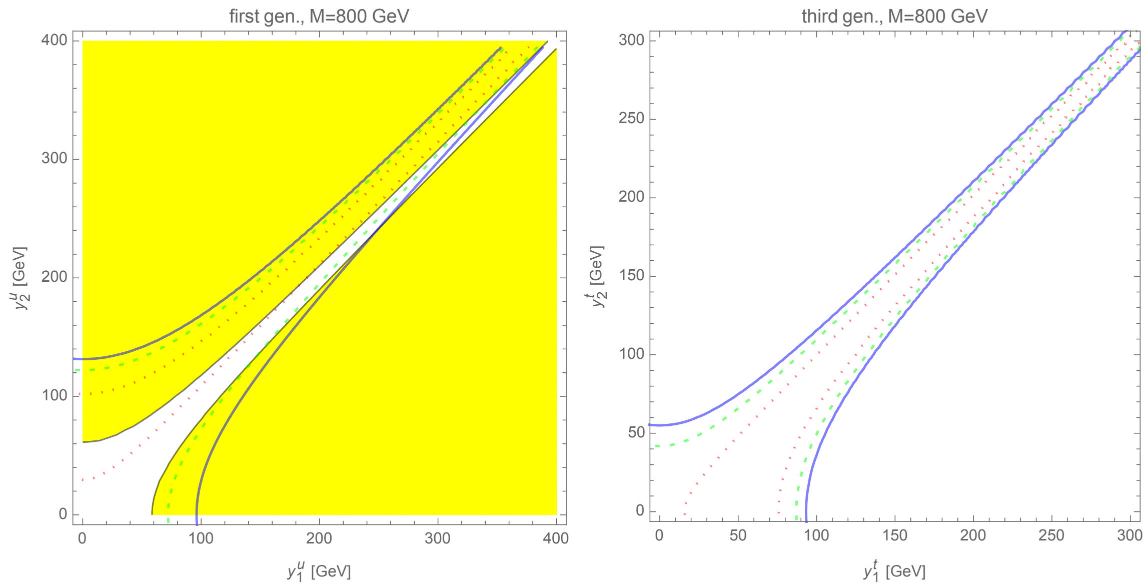

Figure 6. <u>**Doublet $Y = 1/6$ and Doublet $Y = 7/6$**</u> (Section 3.4): EWP bounds at 1σ (red-dashed), 2σ (green-dashed) and 3 σ (blue) for VL quarks coupling with the first (left panel) and third (right panel) SM generations, compared with the region excluded at 3σ by tree-level bounds (yellow region in the left panel). $M = 800$ GeV, $\omega = \omega' = 0$.

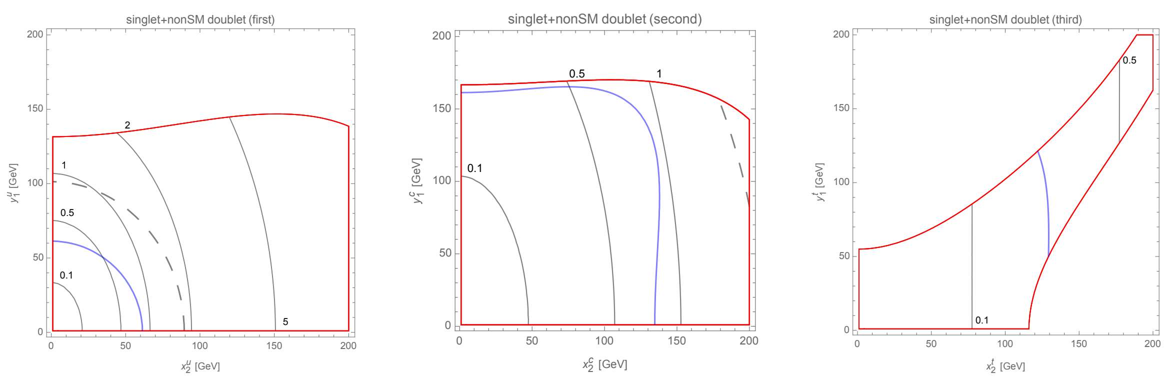

Figure 7. **Singlet $Y = 2/3$ and Doublet $Y = 7/6$** for mixing with first generation only (left), second generation only (middle), third generation (right), and for a mass of the VL quarks of 800 GeV. The channel is T+jet. The grey contour lines correspond to cross-section values in picobarns at 14 TeV. The region inside the red line is allowed by the S and T parameters. The region inside the blue line is allowed by the tree-level bounds. The dashed black lines are the bounds from the ATLAS search [57].

multiplets. The dashed black lines are the bounds at 3σ derived by reinterpreting the results of the ATLAS search [57]. The grey lines represent the contours of the LHC production

cross-section (in pb) for the process of single production of a VL top quark in association with a light jet ($pp \to Tj$).

The results of the "Singlet ($Y = 2/3$) and Doublet ($Y = 7/6$)" scenario are summarised in Figure 7. For this scenario, the tree-level bounds are the most stringent ones if VL quarks mix to the light generations, and they are stronger than the current bound form the ATLAS search. In both cases, the largest $Tjet$ cross section allowed is between 0.5 and 1 pb. In the case of mixing to the third generation only EWP and tree level bounds conspire to select small mixing, and the single production at 14 TeV is limited to small values around 100 fb. Figure 8 shows the results for the "Doublet ($Y = 7/6$) and Triplet ($Y = 5/3$)" scenario. In this case the oblique parameters constrain a much larger region of parameter space and this is due to the much richer exotic quark (quarks with charges 5/3 and 8/3) spectrum, that contributes to the corrections to the oblique parameters. The cross sections for the $tjet$ channel are much smaller than in the previous case, with value around 100 fb for mixing to light generations: this is due to the suppression in the coupling of the t' (belonging to a doublet) to the W boson and a SM down-type quark. For the same reason, in the case of coupling to third generation only, the channel is nearly absent. Figure 9 refers to the "Singlet ($Y = 2/3$) and Doublet ($Y = 1/6$)" scenario. Due to the presence of a SM type VL doublet, this scenario contains right handed charged gauge boson couplings which give additional contributions to the oblique parameters. However, the tree-level constraints are the most stringent for the parameter space of this scenario. We see that in the case of mixing to the first generation, large production rates are allowed, with cross sections above 1 pb and a region already probed by the ATLAS search. Smaller cross sections are attained in the case of mixing to the second generation, while the maximum values drop to about 100 fb for mixing to the third generation. This is the scenario than offers the largest single-production cross sections, and it is a golden case to be studied at the Run 2 of the LHC. Finally, in Figure 10 the results for the "Doublet ($Y = 1/6$) and Doublet ($Y = 7/6$)" scenario are presented. Again, though the presence of an exotic quark with charge 5/3 and of right-handed charged currents which contribute to the corrections to the oblique parameters, the tree-level constraints are stronger for the most part of the parameter space. The large mixings allowed in the cancellation region produce very large $Tjet$ cross sections, with the largest mixing already excluded by the ATLAS search. This is indeed a case where single production can be the most promising channel for the observation of the VL quarks. In the case of mixing the third generation, the single production vanishes: the reason for this is that t's belonging to doublets have very suppressed couplings to the Wb, thus they cannot be produced in association with a light jet but only in association with tops.

An interesting common feature of all the scenario considered is that the LHC bounds can be competitive if not stronger than the tree- and loop-level bounds. Indeed the current LHC data we have considered are already able to constrain region of parameter space otherwise allowed by other observables.

Another type of constraints can be obtained exploiting tools for the recasting of experimental searches for pair production of VL quarks. Considering the bounds on masses and

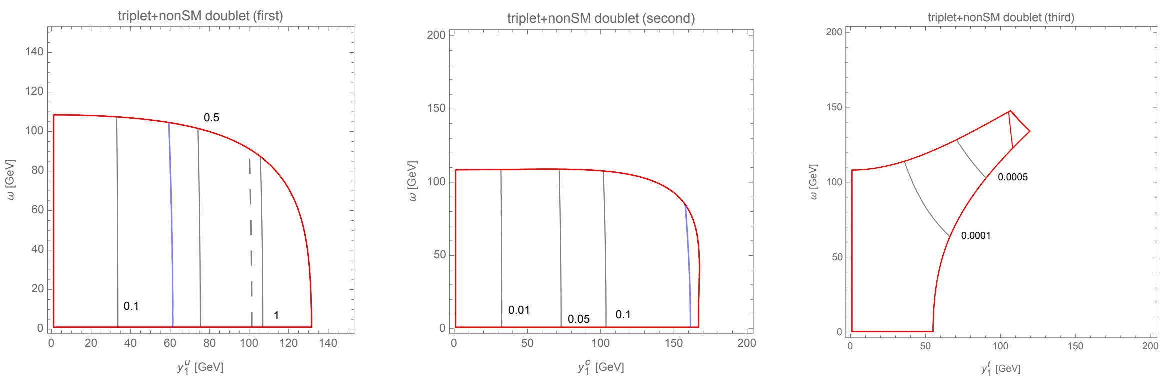

Figure 8. **Doublet $Y = 7/6$ and Triplet $Y = 5/3$** for mixing with first generation only (left) ,second generation only (middle), third generation (right), and for a mass of the VL quarks of 800 GeV. The channel is T+jet. The grey contour lines correspond to cross-section values in picobarns at 14 TeV. The region inside the red line is allowed by the S and T parameters. The region inside the blue line is allowed by the tree-level bounds. The dashed black lines are the bounds from the ATLAS search [57].

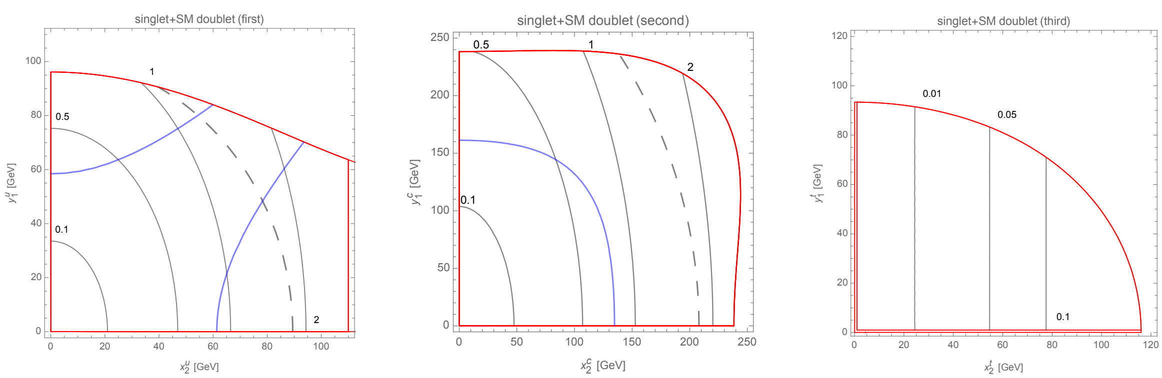

Figure 9. **Singlet $Y = 2/3$ and Doublet $Y = 1/6$** for mixing with first generation only (left), second generation only (middle), third generation (right), and for a mass of the VL quarks of 800 GeV. The channel is T+jet. The grey contour lines correspond to cross-section values in picobarns at 14 TeV. The region inside the red line is allowed by the S and T parameters. The region inside the blue line is allowed by the tree-level bounds. The dashed black lines are the bounds from the ATLAS search [57].

couplings of the VL multiplets it is possible to compute their branching ratios into SM states, and through the recently developed software XQCAT [34, 59, 60], one can determine the exclusion regions by considering results from dedicated searches in pair production and other searches not specifically designed for VL quarks (such as SUSY analyses). This study can be performed systematically for different combinations of VL multiplets, and we postpone this analysis to a subsequent paper, where we will compare the bounds obtained by XQCAT with

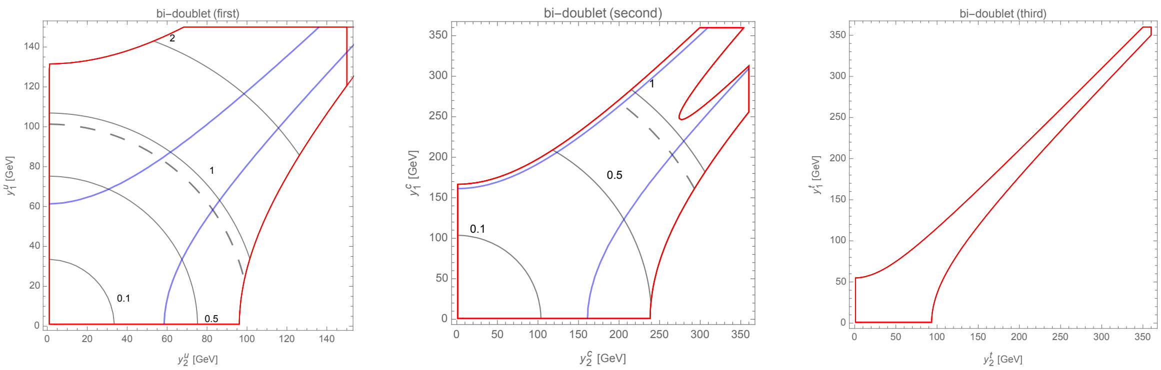

Figure 10. **Doublet $Y = 1/6$ and Doublet $Y = 7/6$** for mixing with first generation only (left), second generation only (middle), third generation (right), and for a mass of the VL quarks of 800 GeV. The channel is T+jet. The grey contour lines correspond to cross-section values in picobarns at 14 TeV (this channel is not allowed in the case of the plot on the right). The region inside the red line is allowed by the S and T parameters. The region inside the blue line is allowed by the tree-level bounds. The dashed black lines are the bounds from the ATLAS search [57].

dedicated simulations for specific scenarios.

6 Conclusions

Vector-like quarks are predicted by many theoretically motivated models of new physics. In most of these models VL quarks appear in complete multiplets and, usually, more than one multiplet is predicted. In this analysis we have considered scenarios with multiple VL quarks both from the point of view of the general mixing structure with the three Standard Model generations and considering the mixing pattern of these multiplets for the determination of mixing effects and precision electroweak observables both at tree-level and at loop-level. The specific case of two different vector-like quark multiplets has been studied in detail, with a special focus on multiplets containing a top partner. The main result of our analysis is that tree-level and loop-level constraints provide complementary information. Moreover the interplay of the vector-like multiplets among themselves and with the Standard model quarks have important consequences for phenomenology as in some cases large single production cross-sections are possible and coupling with light generations is not necessarily suppressed. These results have phenomenological implications for LHC searches as the bounds we have extracted pinpoint particular regions of the parameter space and suggest that in realistic cases containing multiple multiplets of vector-like quarks, cancellations are possible from tree-level bounds which allow large values of the mixing parameters. Even if the EWP tests partially allow to limit these regions where cancellations occur, one has to keep in mind that these loop-level constraints are valid under the assumption that no other states apart from the vector-like multiplets contribute to the S and T parameters. Therefore it is clear that direct searches by the LHC experimental collaborations in the next run of the LHC will play a

mayor role in constraining or discovering physics beyond the Standard Model which contains vector-like multiplets.

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Appendices

A Lagrangian and mass matrices with two VL multiples

The cases are classified into following four categories :

- **Top type multiples:** Only have 2 top VL quarks and no bottom VL quark.
- **Bottom type multiplets:** Only have 2 bottom VL quarks and no top VL quark.
- **Hybrid multiplets:** mixing of top and bottom VL quarks with SM quarks are independent. Hence one can safely take mixing of bottom VL quarks with SM bottom sector to be zero to satisfy all the flavour physics bounds without affecting the top sector.
- **Mixed multiplets:** Remaining cases.

A.1 Top multiplets

In this appendix we consider the multiplets that are presented in the list of Tables 1, 2 and 3. The multiplets containing a top type partner but without any down type partner, in terms of the $(SU(2)_L, U(1)_Y)$ quantum numbers, are the singlet $(\mathbf{1}, 2/3)$, the doublet $(\mathbf{2}, 7/6)$, and the triplet $(\mathbf{3}, 5/3)$, all listed in Table 1.

A.1.1 Singlet $Y = 2/3$ and Doublet $Y = 7/6$

Details are given in Section 3.1.

A.1.2 Doublet $Y = 7/6$ and Triplet $Y = 5/3$

Details are given in Section 3.2.

A.1.3 Singlet $Y = 2/3$ and Triplet $Y = 5/3$

$$\mathcal{L}_{V-SM} = -\lambda_2^k \bar{Q}_L^k \tilde{H} \psi_{2R} + h.c.\,, \tag{A.1}$$

where $\psi_1 = (\mathbf{3}, \frac{5}{3}) = \left(X_1^{8/3}, X_1^{5/3}, U_1\right)^T$ and $\psi_2 = (\mathbf{1}, \frac{2}{3}) = U_2$. The mass lagrangian and mass matrices are:

$$\mathcal{L}_{mass} = -x_2^k u_L^k U_{2R} - M_1 \bar{X}_{1L}^{8/3} X_{1R}^{8/3} - M_1 \bar{X}_{1L}^{5/3} X_{1R}^{5/3} - M_1 \bar{U}_{1L} U_{1R} - M_2 \bar{U}_{2L} U_{2R} + h.c.\,, \tag{A.2}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & 0_{3\times1} & \left(x_2^k\right)_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ 0_{1\times3} & 0 & M_2 \end{pmatrix}, \quad M_{X^{5/3}} = M_1, \quad M_{X^{8/3}} = M_1\,, \tag{A.3}$$

where $\tilde{m}^{up}$ is the SM 3×3 mass matrix of the up sector.

A.2 Bottom multiplets

The multiplets which do not contain a top type partner, but do contain a down type partner, in terms of the $(SU(2)_L, U(1)_Y)$ quantum numbers, are the singlet $(\mathbf{1}, -\frac{1}{3})$ and the doublet $(\mathbf{2}, -\frac{5}{6})$ and triplet $(\mathbf{3}, -\frac{4}{3})$. All these multiplets are listed in Table 2.

A.2.1 Singlet $Y = -1/3$ and Doublet $Y = -5/6$

$$\begin{aligned} \mathcal{L}_{V-SM} &= -\lambda_{1d}^k \, \bar{\psi}_{1L} \tilde{H} d_R^k + h.c.\,, && \text{(A.4)} \\ \mathcal{L}_{V-V} &= -\xi_1 \, \bar{\psi}_{1L} \tilde{H} \psi_{2R} - \xi_2 \, \bar{\psi}_{1R} \tilde{H} \psi_{2L} + h.c.\,, && \text{(A.5)} \end{aligned}$$

where $\psi_1 = (\mathbf{2}, -\frac{5}{6}) = \left(D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{1}, -\frac{1}{3}) = D_2$. The mass lagrangian and mass matrices are:

$$\begin{aligned} \mathcal{L}_{\text{mass}} = &-y_{1d}^k \bar{D}_{1L} d_R^k - \omega \bar{D}_{1L} D_{2R} - \omega' \bar{D}_{1R} D_{2L} - M_1 \, \bar{D}_{1L} D_{1R} - M_1 \, \bar{Y}_{1L} Y_{1R} \\ &-M_2 \, \bar{D}_{2L} D_{2R} + h.c.\,, \end{aligned} \tag{A.6}$$

$$M_u = (\tilde{m}^{up})_{3\times3}\,, \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times3} & 0_{3\times1} & 0_{3\times1} \\ (y_{1d}^k)_{1\times3} & M_1 & \omega \\ 0_{1\times3} & \omega' & M_2 \end{pmatrix}, \quad M_{Y^{-4/3}} = M_1\,, \tag{A.7}$$

where $\tilde{m}^{up}$ and $\tilde{m}^{down}$ are the SM 3×3 mass matrices of the up and down sectors respectively.

A.2.2 Doublet $Y = -5/6$ and triplet $Y = -4/3$

$$\begin{aligned} \mathcal{L}_{V-SM} &= -\lambda_{1d}^k \, \bar{\psi}_{1L} \tilde{H} d_R^k + h.c.\,, && \text{(A.8)} \\ \mathcal{L}_{V-V} &= -\xi_1 \, \bar{\psi}_{1L} \tau^a H (\psi_{2R})^a - \xi_2 \, \bar{\psi}_{1R} \tau^a H (\psi_{2L})^a + h.c.\,, && \text{(A.9)} \end{aligned}$$

where $\psi_1 = (\mathbf{2}, -\frac{5}{6}) = \left(D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{3}, -\frac{4}{3}) = \left(D_2, Y_2^{-4/3}, Y_2^{-7/3}\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = & -y_{1d}^k \bar{D}_{1L} d_R^k - \sqrt{2}\,\omega \bar{D}_{1L} D_{2R} - \omega \bar{Y}_{1L}^{-4/3} Y_{2R}^{-4/3} - \sqrt{2}\,\omega' \bar{D}_{1R} D_{2L} - \omega' \bar{Y}_{1R}^{-4/3} Y_{2L}^{-4/3} \\ & - M_1\, \bar{D}_{1L} D_{1R} - M_1\, \bar{Y}_{1L}^{-4/3} Y_{1R}^{-4/3} - M_2\, \bar{D}_{2L} D_{2R} - M_2\, \bar{Y}_{2L}^{-4/3} Y_{2R}^{-4/3} \\ & - M_2\, \bar{Y}_{2L}^{-7/3} Y_{2R}^{-7/3} + h.c.\,,\end{aligned} \tag{A.10}$$

$$M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times 3} & 0_{3\times 1} & 0_{3\times 1} \\ (y_{1d}^k)_{1\times 3} & M_1 & \sqrt{2}\,\omega \\ 0_{1\times 3} & \sqrt{2}\,\omega' & M_2 \end{pmatrix}, \quad M_{Y^{-4/3}} = \begin{pmatrix} M_1 & \omega \\ \omega' & M_2 \end{pmatrix}, \quad M_{Y^{-7/3}} = M_2. \tag{A.11}$$

A.3 Hybrid multiplets

Multiplets where the mixing parameters of top and bottom sectors are independent. Hence one can evade the constraints coming from b-sector by assuming mixing to be zero without effecting top sector.

A.3.1 SM Doublet $Y = 1/6$ and Singlet $Y = 2/3$

Details are given in Section 3.3.

A.3.2 SM Doublet $Y = 1/6$ and Doublet $Y = 7/6$

Details are given in Section 3.4.

A.3.3 SM Doublet $Y = 1/6$ and Singlet $Y = -1/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k\, \bar{\psi}_{1L} \tilde{H} u_R^k - \lambda_{1d}^k\, \bar{\psi}_{1L} H d_R^k - \lambda_{2d}^k \bar{Q}_L^k H \psi_{2R} + h.c.\,, \tag{A.12}$$

$$\mathcal{L}_{V-V} = -\xi_1\, \bar{\psi}_{1L} H \psi_{2R} - \xi_2\, \bar{\psi}_{1R} H \psi_{2L} + h.c.\,, \tag{A.13}$$

where $\psi_1 = (\mathbf{2}, \frac{1}{6}) = (U_1, D_1)^T$ and $\psi_2 = (\mathbf{1}, -\frac{1}{3}) = D_2$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\text{mass}} = & -y_1^k \bar{U}_{1L} u_R^k - y_{1d}^k \bar{D}_{1L} d_R^k - x_{2d}^k d_L^k D_{2R} - \omega \bar{D}_{1L} D_{2R} - \omega' \bar{D}_{2L} D_{1R} \\ & - M_1\, \bar{D}_{1L} D_{1R} - M_2\, \bar{D}_{2L} D_{2R} - M_1\, \bar{U}_{1L} U_{1R} + h.c.\,,\end{aligned} \tag{A.14}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times 3} & 0_{3\times 1} \\ (y_1^k)_{1\times 3} & M_1 \end{pmatrix}, \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times 3} & 0_{3\times 1} & (x_{2d}^k)_{3\times 1} \\ (y_{1d}^k)_{1\times 3} & M_1 & \omega \\ 0_{1\times 3} & \omega' & M_2 \end{pmatrix}. \tag{A.15}$$

A.3.4 SM Doublet $Y = 1/6$ and Doublet $Y = -5/6$

$$\mathcal{L}_{V-SM} = -\lambda_1^k \bar{\psi}_{1L}\tilde{H}u_R^k - \lambda_{1d}^k \bar{\psi}_{1L}Hd_R^k - \lambda_{2d}^k\bar{\psi}_{2L}\tilde{H}d_R^k + h.c.\,, \tag{A.16}$$

where $\psi_1 = (\mathbf{2}, \frac{1}{6}) = (U_1, D_1)^T$ and $\psi_2 = (\mathbf{2}, -\frac{5}{6}) = \left(D_2, Y_2^{-4/3}\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\text{mass}} =& -y_1^k\bar{U}_{1L}u_R^k - y_{1d}^k\bar{D}_{1L}d_R^k - y_{2d}^k\bar{D}_{2L}d_R^k - M_1\,\bar{U}_{1L}U_{1R} - M_1\,\bar{D}_{1L}D_{1R}\\ &-M_2\,\bar{D}_{2L}D_{2R} - M_2\,\bar{Y}_{2L}^{-4/3}Y_{2R}^{-4/3} + h.c.\,,\end{aligned} \tag{A.17}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & 0_{3\times1} \\ (y_1^k)_{1\times3} & M_1 \end{pmatrix}\,, \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & 0_{3\times1} & 0_{3\times1} \\ (y_{1d}^k)_{1\times3} & M_1 & 0 \\ (y_{2d}^k)_{1\times3} & 0 & M_2 \end{pmatrix}\,, \quad M_{Y^{-4/3}} = M_2\,. \tag{A.18}$$

A.4 Mixed multiplets

The remaining combinations contain multiplets with both a VL top partner and a VL bottom partner but with non-independent mixing in the up and in the down sector. They are listed in Table 3. These combinations are not considered in our numerical studies, however their mixing structure with the SM and the other VL multiplets is described in the following.

A.4.1 SM Doublet $Y = 1/6$ and Triplet $Y = 2/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k \bar{\psi}_{1L}\tilde{H}u_R^k - \lambda_{1d}^k \bar{\psi}_{1L}Hd_R^k - \lambda_2^k\,\bar{Q}_L^k\tilde{H}\tau^a\psi_{2R}^a + h.c.\,, \tag{A.19}$$

$$\mathcal{L}_{V-V} = -\xi_1\,\bar{\psi}_{1L}\tilde{H}\tau^a\psi_{2R}^a - \xi_2\,\bar{\psi}_{1R}\tilde{H}\tau^a\psi_{2L}^a + h.c.\,, \tag{A.20}$$

where $\psi_1 = (\mathbf{2}, 1/6) = (U_1, D_1)^T$ and $\psi_2 = \left(X_2^{5/3}, U_2, D_2\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\text{mass}} =& -y_1^k\bar{U}_{1L}u_R^k - y_{1d}^k\bar{D}_{1L}d_R^k - x_2^k\left(\bar{u}_L^kU_{2R} + \sqrt{2}\,\bar{d}_L^kD_{2R}\right) - \omega\left(\bar{U}_{1L}U_{2R} + \sqrt{2}\bar{D}_{1L}D_{2R}\right)\\ &-\omega'\left(\bar{U}_{2L}U_{1R} + \sqrt{2}\bar{D}_{2L}D_{1R}\right) - M_1\,\bar{U}_{1L}U_{1R} - M_1\,\bar{D}_{1L}D_{1R} - M_2\,\bar{U}_{2L}U_{2R}\\ &-M_2\,\bar{D}_{2L}D_{2R} - M_2\,\bar{X}_{2L}^{5/3}X_{2R}^{5/3} + h.c.\,,\end{aligned} \tag{A.21}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & 0_{3\times1} & (x_2^k)_{3\times1} \\ (y_1^k)_{1\times3} & M_1 & \omega \\ 0_{1\times3} & \omega' & M_2 \end{pmatrix}\,, \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & 0_{3\times1} & \sqrt{2}\,(x_2^k)_{3\times1} \\ (y_{1d}^k)_{1\times3} & M_1 & \sqrt{2}\,\omega \\ 0_{1\times3} & \sqrt{2}\,\omega' & M_2 \end{pmatrix}\,, \quad M_{X^{5/3}} = M_2\,. \tag{A.22}$$

A.4.2 SM Doublet $Y = 1/6$ and Triplet $Y = -1/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k \bar{\psi}_{1L}\tilde{H}u_R^k - \lambda_{1d}^k \bar{\psi}_{1L}Hd_R^k - \lambda_2^k\,\bar{Q}_L^kH\tau^a\psi_{2R}^a + h.c.\,, \tag{A.23}$$

$$\mathcal{L}_{V-V} = -\xi_1\,\bar{\psi}_{1L}H\tau^a\psi_{2R}^a - \xi_2\,\bar{\psi}_{1R}H\tau^a\psi_{2L}^a + h.c.\,, \tag{A.24}$$

where $\psi_1 = (\mathbf{2}, \frac{1}{6}) = (U_1, D_1)^T$ and $\psi_2 = (\mathbf{3}, -\frac{1}{3}) = \left(U_2, D_2, Y_2^{-4/3}\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\rm mass} \;=\; & -y_1^k \bar{U}_{1L} u_R^k - y_{1d}^k \bar{D}_{1L} d_R^k - x_2^k\left(\sqrt{2}\,\bar{u}_L^k U_{2R} - \bar{d}_L^k D_{2R}\right) - \omega\left(\sqrt{2}\,\bar{U}_{1L} U_{2R} - \bar{D}_{1L} D_{2R}\right)\\ & -\omega'\left(\sqrt{2}\,\bar{U}_{2L} U_{1R} - \bar{D}_{2L} D_{1R}\right) - M_1\,\bar{U}_{1L} U_{1R} - M_1\,\bar{D}_{1L} D_{1R} - M_2\,\bar{U}_{2L} U_{2R}\\ & -M_2\,\bar{D}_{2L} D_{2R} - M_2\,\bar{X}_{2L}^{5/3} X_{2R}^{5/3} + h.c.\,,\end{aligned} \tag{A.25}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & 0_{3\times1} & \sqrt{2}\,(x_2^k)_{3\times1} \\ (y_1^k)_{1\times3} & M_1 & \sqrt{2}\,\omega \\ 0_{1\times3} & \sqrt{2}\,\omega' & M_2 \end{pmatrix}, \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & 0_{3\times1} & -(x_2^k)_{3\times1} \\ (y_{1d}^k)_{1\times3} & M_1 & -\omega \\ 0_{1\times3} & -\omega' & M_2 \end{pmatrix}, \quad M_{X^{5/3}} = M_2\,. \tag{A.26}$$

A.4.3 Triplet $Y = 2/3$ and Singlet $Y = 2/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k\,\bar{Q}_L \tilde{H} \psi_{1R} - \lambda_2^k\,\bar{Q}_L^k \tilde{H} \tau^a \psi_{2R}^a + h.c.\,, \tag{A.27}$$

where $\psi_1 = (\mathbf{1}, \frac{2}{3}) = U_1$ and $\psi_2 = (\mathbf{3}, \frac{2}{3}) = \left(U_2, D_2, Y_2^{-4/3}\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\rm mass} \;=\; & -x_1^k \bar{u}_L^k U_{1R} - x_2^i\left(\bar{u}_L^i U_{2R} + \sqrt{2}\,\bar{d}_L^i D_{2R}\right)\\ & -M_1\,\bar{U}_{1L} U_{1R} - M_2\,\bar{U}_{2L} U_{2R} - M_2\,\bar{D}_{2L} D_{2R} - M_2\,\bar{Y}_{2L}^{-4/3} Y_{2R}^{-4/3} + h.c.\,,\end{aligned} \tag{A.28}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & (x_1^k)_{3\times1} & (x_2^k)_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ 0_{1\times3} & 0 & M_2 \end{pmatrix}, \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & \sqrt{2}\,(x_2^k)_{3\times1} \\ 0_{1\times3} & M_2 \end{pmatrix}, \quad M_{Y^{-4/3}} = M_2\,. \tag{A.29}$$

A.4.4 Triplet $Y = 2/3$ and Doublet $Y = 7/6$

$$\mathcal{L}_{V-SM} \;=\; -\lambda_1^k\,\bar{\psi}_{1L} H u_R^k - \lambda_2^k\,\bar{Q}_L^k \tilde{H} \tau^a \psi_{2R}^a + h.c.\,, \tag{A.30}$$

$$\mathcal{L}_{V-V} \;=\; -\xi_1\,\bar{\psi}_{1L} H \tau^a \psi_{2R}^a - \xi_2\,\bar{\psi}_{1R} H \tau^a \psi_{2L}^a + h.c.\,, \tag{A.31}$$

where $\psi_1 = (\mathbf{2}, \frac{7}{6}) = \left(X_1^{5/3}, U_1\right)^T$ and $\psi_2 = (\mathbf{3}, \frac{2}{3}) = \left(X_2^{5/3}, U_2, D_2\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}\mathcal{L}_{\rm mass} \;=\; & -y_1^k \bar{U}_{1L} u_R^k - x_2^i\left(\bar{u}_L^i U_{2R} + \sqrt{2}\,\bar{d}_L^i D_{2R}\right) - \omega\left(\sqrt{2}\,\bar{X}_{1L}^{5/3} X_{2R}^{5/3} - \bar{U}_{1L} U_{2R}\right)\\ & -\omega'\left(\sqrt{2}\,\bar{X}_{2L}^{5/3} X_{1R}^{5/3} - \bar{U}_{2L} U_{1R}\right) - M_1\,\bar{U}_{1L} U_{1R} - M_1\,\bar{X}_{1L}^{5/3} X_{1R}^{5/3} - M_2\,\bar{X}_{2L}^{5/3} X_{2R}^{5/3}\\ & -M_2\,\bar{U}_{2L} U_{2R} - M_2\,\bar{D}_{2L} D_{2R} + h.c.\,,\end{aligned} \tag{A.32}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & 0_{3\times1} & (x_2^k)_{3\times1} \\ (y_1^k)_{1\times3} & M_1 & -\omega \\ 0_{1\times3} & -\omega' & M_2 \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & \sqrt{2}\,(x_2^k)_{3\times1} \\ 0_{1\times3} & M_2 \end{pmatrix}$$

$$M_{Y^{-4/3}} = \begin{pmatrix} M_1 & \sqrt{2}\,\omega \\ \sqrt{2}\,\omega' & M_2 \end{pmatrix} . \tag{A.33}$$

A.4.5 Triplet $Y = 2/3$ and Singlet $Y = -1/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k\,\bar{Q}_L^k \tilde{H}\tau^a\psi_{1R}^a - \lambda_{2d}^k\,\bar{Q}_L^k H\psi_{2R} + h.c.\,, \tag{A.34}$$

where $\psi_1 = (\mathbf{3}, \frac{2}{3}) = \left(X_1^{5/3}, U_1, D_1\right)^T$ and $\psi_2 = (\mathbf{1}, -\frac{1}{3}) = D_2$. The mass lagrangian and mass matrices are:

$$\begin{aligned} \mathcal{L}_{\rm mass} = & -x_1^k\left(\bar{u}_L^k U_{1R} + \sqrt{2}\,\bar{d}_L^k D_{1R}\right) - x_{2d}^k\,\bar{d}_L^k D_{2R} - M_1\,\bar{X}_{1L}^{5/3}X_{1R}^{5/3} - M_1\,\bar{U}_{1L}U_{1R} \\ & -M_1\,\bar{D}_{1L}D_{1R} - M_2\,\bar{D}_{2L}D_{2R} + h.c.\,, \end{aligned} \tag{A.35}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & (x_1^k)_{3\times1} \\ 0_{1\times3} & M_1 \end{pmatrix} , \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & \sqrt{2}\,(x_1^k)_{3\times1} & (x_{2d}^k)_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ 0_{1\times3} & 0 & M_2 \end{pmatrix} , \quad M_{X^{5/3}} = M_1\,. \tag{A.36}$$

A.4.6 Triplet $Y = 2/3$ and Doublet $Y = -5/6$

$$\mathcal{L}_{V-SM} = -\lambda_1^k\,\bar{Q}_L^k \tilde{H}\tau^a\psi_{1R}^a - \lambda_{2d}^k\,\bar{\psi}_{2L}\tilde{H}d_R^k + h.c.\,, \tag{A.37}$$

where $\psi_1 = (\mathbf{3}, 2/3) = \left(X_1^{5/3}, U_1, D_1\right)^T$ and $\psi_2 = (\mathbf{2}, -5/6) = \left(D_2, Y_2^{-4/3}\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned} \mathcal{L}_{\rm mass} = & -x_1^k\left(\bar{u}_L^k U_{1R} + \sqrt{2}\,\bar{d}_L^k D_{1R}\right) - y_{2d}^k\,\bar{D}_{2L}d_R^k - M_1\,\bar{X}_{1L}^{5/3}X_{1R}^{5/3} - M_1\,\bar{U}_{1L}U_{1R} \\ & -M_1\,\bar{D}_{1L}D_{1R} - M_2\,\bar{D}_{2L}D_{2R} + h.c.\,, \end{aligned} \tag{A.38}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & (x_1^k)_{3\times1} \\ 0_{1\times3} & M_1 \end{pmatrix} , \quad M_d = \begin{pmatrix} (\tilde{m}^{down})_{3\times3} & \sqrt{2}\,(x_1^k)_{3\times1} & 0_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ (y_{2d}^k)_{1\times3} & 0 & M_2 \end{pmatrix} , \quad M_{X^{5/3}} = M_1\,. \tag{A.39}$$

A.4.7 Triplet $Y = -1/3$ and Singlet $Y = 2/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k\,\bar{Q}_L^k H\tau^a\psi_{1L}^a - \lambda_2^k\,\bar{Q}_L^k \tilde{H}\psi_{2R} + h.c.\,, \tag{A.40}$$

where $\psi_1 = (\mathbf{3}, -1/3) = \left(U_1, D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{1}, 2/3) = U_2$. The mass lagrangian and mass matrices are:

$$\begin{aligned} \mathcal{L}_{\rm mass} = & -x_1^k\left(\sqrt{2}\,\bar{u}_L^k U_{1R} - \bar{d}_L^k D_{1R}\right) - x_2^k\bar{u}_L^k U_{2R} - M_1\,\bar{U}_{1L}U_{1R} - M_1\,\bar{D}_{1L}D_{1R} \\ & -M_1\,\bar{Y}_{1L}^{-4/3}Y_{1R}^{-4/3} - M_2\,\bar{U}_{2L}U_{2R} + h.c.\,, \end{aligned} \tag{A.41}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & \sqrt{2}\,(x_1^k)_{3\times1} & (x_2^k)_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ 0_{1\times3} & 0 & M_2 \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times3} & -(x_1^k)_{3\times1} \\ 0_{1\times3} & M_1 \end{pmatrix} ,$$
$$M_{Y^{-4/3}} = M_1 \,. \tag{A.42}$$

A.4.8 Triplet $Y = -1/3$ and Doublet $Y = 7/6$

$$\mathcal{L}_{V-SM} = -\lambda_1^k \, \bar{Q}_L^k H \tau^a \psi_{1L}^a - \lambda_2^k \, \bar{Q}_L^k \tilde{H} \psi_{2R} + h.c. \,, \tag{A.43}$$

where $\psi_1 = (\mathbf{3}, -1/3) = \left(U_1, D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{2}, 7/6) = \left(X_2^{5/3}, U_2\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned} \mathcal{L}_{\rm mass} = & -x_1^k \left(\sqrt{2}\,\bar{u}_L^k U_{1R} - \bar{d}_L^k D_{1R}\right) - x_2^k \, \bar{u}_L^k U_{2R} - M_1 \, \bar{U}_{1L} U_{1R} - M_1 \, \bar{D}_{1L} D_{1R} \\ & - M_1 \, \bar{Y}_{1L}^{-4/3} Y_{1R}^{-4/3} - M_2 \, \bar{U}_{2L} U_{2R} + h.c. \,, \end{aligned} \tag{A.44}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & \sqrt{2}\,(x_1^k)_{3\times1} & (x_2^k)_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ 0_{1\times3} & 0 & M_2 \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times3} & -(x_1^k)_{3\times1} \\ 0_{1\times3} & M_1 \end{pmatrix} ,$$
$$M_{Y^{-4/3}} = M_1 \,. \tag{A.45}$$

A.4.9 Triplet $Y = -1/3$ and Singlet $Y = -1/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k \, \bar{Q}_L^k H \tau^a \psi_{1L}^a - \lambda_{2d}^k \, \bar{Q}_L^k H \psi_{2R} + h.c. \,, \tag{A.46}$$

where $\psi_1 = (\mathbf{3}, -1/3) = \left(U_1, D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{1}, -1/3) = D_2$. The mass lagrangian and mass matrices are:

$$\begin{aligned} \mathcal{L}_{\rm mass} = & -x_1^k \left(\sqrt{2}\,\bar{u}_L^k U_{1R} - \bar{d}_L^k D_{1R}\right) - x_{2d}^k \, \bar{d}_L^k D_{2R} - M_1 \, \bar{U}_{1L} U_{1R} - M_1 \, \bar{D}_{1L} D_{1R} \\ & - M_1 \, \bar{Y}_{1L}^{-4/3} Y_{1R}^{-4/3} - M_2 \, \bar{D}_{2L} D_{2R} + h.c. \,, \end{aligned} \tag{A.47}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times3} & \sqrt{2}\,(x_1^k)_{3\times1} \\ 0_{1\times3} & M_1 \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times3} & -(x_1^k)_{3\times1} & (x_{2d}^k)_{3\times1} \\ 0_{1\times3} & M_1 & 0 \\ 0_{1\times3} & 0 & M_2 \end{pmatrix} ,$$
$$M_{Y^{-4/3}} = M_1 \,. \tag{A.48}$$

A.4.10 Triplet $Y = -1/3$ and Doublet $Y = -5/6$

$$\mathcal{L}_{V-SM} = -\lambda_{1d}^k \, \bar{\psi}_{1L} \tilde{H} d_R^k - \lambda_2^k \, \bar{Q}_L^k H \tau^a \psi_{2L}^a + h.c. \,, \tag{A.49}$$
$$\mathcal{L}_{V-V} = -\xi_1 \, \bar{\psi}_{1L} \tilde{H} \tau^a \psi_{2R}^a - \xi_2 \, \bar{\psi}_{1R} \tilde{H} \tau^a \psi_{2L}^a + h.c. \,, \tag{A.50}$$

where $\psi_1 = (\mathbf{2}, -5/6) = \left(D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{3}, -1/3) = \left(U_2, D_2, Y_2^{-4/3}\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}
\mathcal{L}_{\text{mass}} = & -y_{1d}^k \bar{D}_{1L} d_R^k - x_2^k \left(\sqrt{2}\, \bar{u}_L^k U_{2R} - \bar{d}_L^k D_{2R}\right) - \omega \left(\bar{D}_{1L} D_{2R} + \sqrt{2}\, \bar{Y}_{1L}^{-4/3} Y_{2R}^{-4/3}\right) \\
& -\omega' \left(\bar{D}_{2L} D_{1R} + \sqrt{2}\, \bar{Y}_{2L}^{-4/3} Y_{1R}^{-4/3}\right) - M_1\, \bar{D}_{1L} D_{1R} - M_1\, \bar{Y}_{1L}^{-4/3} Y_{1R}^{-4/3} \\
& -M_2\, \bar{U}_{2L} U_{2R} - M_2\, \bar{D}_{2L} D_{2R} - M_2\, \bar{Y}_{2L}^{-4/3} Y_{2R}^{-4/3} + h.c.\,,
\end{aligned} \tag{A.51}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times 3} & \sqrt{2}\,(x_2^k)_{3\times 1} \\ 0_{1\times 3} & M_2 \end{pmatrix} \quad , \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times 3} & 0_{3\times 1} & -(x_2^k)_{3\times 1} \\ \left(y_{1d}^k\right)_{1\times 3} & M_1 & \omega \\ 0_{1\times 3} & \omega' & M_2 \end{pmatrix},$$
$$M_{Y^{-4/3}} = \begin{pmatrix} M_1 & \sqrt{2}\,\omega \\ \sqrt{2}\,\omega' & M_2 \end{pmatrix}. \tag{A.52}$$

A.4.11 Triplet $Y = 2/3$ and Triplet $Y = -1/3$

$$\mathcal{L}_{V-SM} = -\lambda_1^k\, \bar{Q}_L^k H \tau^a \psi_{1R}^a - \lambda_2^k\, \bar{Q}_L^k \tilde{H} \tau^a \psi_{2R}^a + h.c.\,, \tag{A.53}$$

where $\psi_1 = (\mathbf{3}, -1/3) = \left(U_1, D_1, Y_1^{-4/3}\right)^T$ and $\psi_2 = (\mathbf{3}, 2/3) = \left(X_2^{5/3}, U_2, D_2\right)^T$. The mass lagrangian and mass matrices are:

$$\begin{aligned}
\mathcal{L}_{\text{mass}} = & -x_1^k \left(\sqrt{2}\, \bar{u}_L^k U_{1R} - \bar{d}_L^k D_{1R}\right) - x_2^k \left(\bar{u}_L^k U_{2R} - \sqrt{2}\, \bar{d}_L^k D_{2R}\right) \\
& -M_1\, \bar{U}_{1L} U_{1R} - M_1\, \bar{D}_{1L} D_{1R} - M_1\, \bar{Y}_{1L}^{-4/3} Y_{1R}^{-4/3} \\
& -M_2\, \bar{U}_{2L} U_{2R} - M_2\, \bar{X}_{2L}^{5/3} X_{2R}^{5/3} - M_2\, \bar{D}_{2L} D_{2R} + h.c.\,,
\end{aligned} \tag{A.54}$$

$$M_u = \begin{pmatrix} (\tilde{m}^{up})_{3\times 3} & \sqrt{2}\,(x_1^k)_{3\times 1} & (x_2^k)_{3\times 1} \\ 0_{1\times 3} & M_1 & 0 \\ 0_{1\times 3} & 0 & M_2 \end{pmatrix}, \quad M_d = \begin{pmatrix} \left(\tilde{m}^{down}\right)_{3\times 3} & -(x_1^k)_{3\times 1} & -\sqrt{2}\,(x_2^k)_{3\times 1} \\ 0_{1\times 3} & M_1 & 0 \\ 0_{1\times 3} & 0 & M_2 \end{pmatrix},$$
$$M_{Y^{-4/3}} = M_1\,, \quad M_{X^{5/3}} = M_2\,. \tag{A.55}$$

B Couplings to gauge and Higgs bosons

The VL quarks couple to gauge bosons and the Higgs boson according to their quantum numbers. In the following we give some general formulas for the case of two VL quarks multiplets which were used for our numerical results and which can be easily generalised for scenarios with more than two VL quark multiplets.

Model	α_1	α_2	$\alpha_1^{X^{5/3}}$	$\alpha_2^{X^{5/3}}$	$\alpha_1^{X^{8/3}}$	$\alpha_2^{X^{8/3}}$	$\alpha_1^{Y^{-4/3}}$	$\alpha_2^{Y^{-4/3}}$	$\alpha_1^{Y^{-7/3}}$	$\alpha_2^{Y^{-7/3}}$
A.1.1	0	0	1	0	0	0	0	0	0	0
A.1.2	0	0	1	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	0	0
A.1.3	0	0	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	0	0	0
A.2.1	0	0	0	0	0	0	1	0	0	0
A.2.2	0	0	0	0	0	0	1	$-\sqrt{2}$	0	$\sqrt{2}$
A.3.1	1	0	0	0	0	0	0	0	0	0
A.3.2	1	0	0	1	0	0	0	0	0	0
A.3.3	1	0	0	0	0	0	0	0	0	0
A.3.4	1	0	0	0	0	0	0	1	0	0
A.4.1	1	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	0	0	0	0
A.4.2	1	$-\sqrt{2}$	0	0	0	0	0	$\sqrt{2}$	0	0
A.4.3	0	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	0	0	0	0
A.4.4	0	$\sqrt{2}$	1	$-\sqrt{2}$	0	0	0	0	0	0
A.4.5	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	0	0	0	0	0
A.4.6	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	0	0	1	0	0
A.4.7	$-\sqrt{2}$	0	0	0	0	0	$\sqrt{2}$	0	0	0
A.4.8	$-\sqrt{2}$	0	0	1	0	0	$\sqrt{2}$	0	0	0
A.4.9	$-\sqrt{2}$	0	0	0	0	0	$\sqrt{2}$	0	0	0
A.4.10	0	$-\sqrt{2}$	0	0	0	0	1	$\sqrt{2}$	0	0
A.4.11	$-\sqrt{2}$	$\sqrt{2}$	0	$-\sqrt{2}$	0	0	$\sqrt{2}$	0	0	0

Table 5. The coefficients α_i, $\alpha_i^{X^{5/3}}$, $\alpha_i^{X^{8/3}}$, $\alpha_i^{Y^{-4/3}}$ and $\alpha_i^{Y^{-7/3}}$ ($i = 1, 2$) in the two vector-like multiplets models listed in Appendix A.

B.1 $W^\pm$ boson couplings

In the gauge basis, the general expressions for the couplings of $W^\pm$ bosons in the two VL multiplets models are given by

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \left(\bar{u}_L^1,\, \bar{u}_L^2,\, \bar{u}_L^3,\, \bar{U}_{1L},\, \bar{U}_{2L} \right) \cdot \delta_L \cdot \gamma^\mu \begin{pmatrix} d_L^1 \\ d_L^2 \\ d_L^3 \\ D_{1L} \\ D_{2L} \end{pmatrix} W_\mu^+$$

$$+ \frac{g}{\sqrt{2}} \left(\bar{u}_R^1,\, \bar{u}_R^2,\, \bar{u}_R^3,\, \bar{U}_{1R},\, \bar{U}_{2R} \right) \cdot \delta_R \cdot \gamma^\mu \begin{pmatrix} d_R^1 \\ d_R^2 \\ d_R^3 \\ D_{1R} \\ D_{2R} \end{pmatrix} W_\mu^+ + h.c.\,, \tag{B.1}$$

with

$$\delta_L = \begin{pmatrix} I_{3\times3} & & \\ & \alpha_1 & \\ & & \alpha_2 \end{pmatrix}, \quad \delta_R = \begin{pmatrix} 0_{3\times3} & & \\ & \alpha_1 & \\ & & \alpha_2 \end{pmatrix}. \tag{B.2}$$

Note that the coefficients α_i, which are listed in Table 5, depend on the representation of the i-th VL quark. In the mass basis, the left- and right-handed couplings can be written as

$$g_{WL}^{IJ} = \frac{g}{\sqrt{2}} V_{CKM}^{L,IJ} = \frac{g}{\sqrt{2}} V_L^{u\dagger} \cdot \delta_L \cdot V_L^d \,, \tag{B.3}$$

$$g_{WR}^{IJ} = \frac{g}{\sqrt{2}} V_{CKM}^{R,IJ} = \frac{g}{\sqrt{2}} V_R^{u\dagger} \cdot \delta_R \cdot V_R^d \,, \tag{B.4}$$

where V_{CKM}^L and V_{CKM}^R are the left- and right-handed CKM matrix, respectively. The Lagrangian terms for the couplings between exotic quark $X^{5/3}/Y^{-4/3}$ and top-type/bottom-type quarks can be expressed as:

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \left(0,\, 0,\, 0,\, \bar{X}_{1L}^{5/3},\, \bar{X}_{2L}^{5/3}\right) \cdot \begin{pmatrix} 0_{3\times3} & & \\ & \alpha_1^{X^{5/3}} & \\ & & \alpha_2^{X^{5/3}} \end{pmatrix} \cdot \gamma^\mu \begin{pmatrix} u_L^1 \\ u_L^2 \\ u_L^3 \\ U_{1L} \\ U_{2L} \end{pmatrix} W_\mu^+ + h.c.\,. \tag{B.5}$$

The mass matrix of the $X^{5/3}$ system is diagonalised as:

$$M_{X^{5/3}} = V_L^{X^{5/3}} \cdot \begin{pmatrix} m_{X_L^{5/3}} & \\ & m_{X_H^{5/3}} \end{pmatrix} \cdot V_R^{X^{5/3}\dagger} \,, \tag{B.6}$$

where the mass eigenstates $X_L^{5/3}$ and $X_H^{5/3}$ are defined as:

$$\begin{pmatrix} X_L^{5/3} \\ X_H^{5/3} \end{pmatrix}_{L/R} = V_{L/R}^{X^{5/3}\dagger} \cdot \begin{pmatrix} X_1^{5/3} \\ X_2^{5/3} \end{pmatrix}_{L/R} . \tag{B.7}$$

In the mass basis, the left- and right-handed couplings of $X^{5/3}$ become:

$$g_{WL}^{X^{5/3},IJ} = \frac{g}{\sqrt{2}} \begin{pmatrix} I_{3\times3} & \\ & V_L^{X^{5/3}} \end{pmatrix}^\dagger \cdot \begin{pmatrix} 0_{3\times3} & & \\ & \alpha_1^{X^{5/3}} & \\ & & \alpha_2^{X^{5/3}} \end{pmatrix} \cdot V_L^u \,, \tag{B.8}$$

$$g_{WR}^{X^{5/3},IJ} = \frac{g}{\sqrt{2}} \begin{pmatrix} I_{3\times3} & \\ & V_R^{X^{5/3}} \end{pmatrix}^\dagger \cdot \begin{pmatrix} 0_{3\times3} & & \\ & \alpha_1^{X^{5/3}} & \\ & & \alpha_2^{X^{5/3}} \end{pmatrix} \cdot V_R^u \,. \tag{B.9}$$

Similarly, the couplings of $Y^{-4/3}$ can be expressed as

$$g_{WL}^{Y^{-4/3},IJ} = \frac{g}{\sqrt{2}} V_L^{d\dagger} \cdot \begin{pmatrix} 0_{3\times 3} & & \\ & \alpha_1^{Y^{-4/3}} & \\ & & \alpha_2^{Y^{-4/3}} \end{pmatrix} \cdot \begin{pmatrix} I_{3\times 3} & \\ & V_L^{Y^{-4/3}} \end{pmatrix}, \tag{B.10}$$

$$g_{WR}^{Y^{-4/3},IJ} = \frac{g}{\sqrt{2}} V_R^{d\dagger} \cdot \begin{pmatrix} 0_{3\times 3} & & \\ & \alpha_1^{Y^{-4/3}} & \\ & & \alpha_2^{Y^{-4/3}} \end{pmatrix} \cdot \begin{pmatrix} I_{3\times 3} & \\ & V_R^{Y^{-4/3}} \end{pmatrix}. \tag{B.11}$$

We also introduce the general expressions for the couplings between $X^{5/3}$ and $X^{8/3}$:

$$\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \begin{pmatrix} \bar{X}_{1L}^{8/3}, \bar{X}_{2L}^{8/3} \end{pmatrix} \cdot \begin{pmatrix} \alpha_1^{X^{8/3}} & \\ & \alpha_2^{X^{8/3}} \end{pmatrix} \cdot \gamma^\mu \begin{pmatrix} X_{1L}^{5/3} \\ X_{2L}^{5/3} \end{pmatrix} W_\mu^+ + h.c.. \tag{B.12}$$

Note that there is no mixing between $X_1^{8/3}$ and $X_2^{8/3}$ in the two VL multiplets listed in Appendix A. The left- and right-handed couplings of $X^{8/3}$ in the mass basis are given by:

$$g_{WL}^{X^{8/3},IJ} = \frac{g}{\sqrt{2}} \begin{pmatrix} \alpha_1^{X^{8/3}} & \\ & \alpha_2^{X^{8/3}} \end{pmatrix} \cdot V_L^{X^{5/3}}, \tag{B.13}$$

$$g_{WR}^{X^{8/3},IJ} = \frac{g}{\sqrt{2}} \begin{pmatrix} \alpha_1^{X^{8/3}} & \\ & \alpha_2^{X^{8/3}} \end{pmatrix} \cdot V_R^{X^{5/3}}. \tag{B.14}$$

For exotic quark $Y^{-7/3}$, the couplings can be evaluated as:

$$g_{WL}^{Y^{-7/3},IJ} = \frac{g}{\sqrt{2}} V_L^{Y^{-4/3}\dagger} \cdot \begin{pmatrix} \alpha_1^{Y^{-7/3}} & \\ & \alpha_2^{Y^{-7/3}} \end{pmatrix}, \tag{B.15}$$

$$g_{WR}^{Y^{-7/3},IJ} = \frac{g}{\sqrt{2}} V_R^{Y^{-4/3}\dagger} \cdot \begin{pmatrix} \alpha_1^{Y^{-7/3}} & \\ & \alpha_2^{Y^{-7/3}} \end{pmatrix}. \tag{B.16}$$

B.2 Z boson couplings

In terms of Z boson couplings to the quark sector, and for the case of two VL quarks mixing with any SM quark generation under consideration, it is possible to identify three scenarios depending on where FCNCs appear. In the Top type multiplets listed in Appendix A.1 FCNCs appear in the up quark sector; in the Bottom type multiplets listed in Appendix A.2 FCNCs appear in the down quark sector; finally, in the Hybrid and Mixed multiplets, Appendix A.3, A.4, FCNCs appear in both sectors.

The general expression for the left-handed couplings of the Z in the up quark sector can be written as:

$$\mathcal{L}_Z = \frac{g}{c_W} \left(\bar{u}_L^1, \bar{u}_L^2, \bar{u}_L^3, \bar{U}_{1L}, \bar{U}_{2L}\right) \cdot \left[\left(\frac{1}{2} - Q_u s_W^2\right) I_{5\times 5} - \Delta T_3^{(up)}\right] \gamma^\mu \cdot \begin{pmatrix} u_L^1 \\ u_L^2 \\ u_L^3 \\ U_{1L} \\ U_{2L} \end{pmatrix} Z_\mu, \tag{B.17}$$

with:

$$\Delta T_3^{(up)} = \begin{pmatrix} 0_{3\times 3} & & \\ & \Delta T_3^{(1,u)} & \\ & & \Delta T_3^{(2,u)} \end{pmatrix}, \tag{B.18}$$

where $I_{5\times5}$ is the 5×5 unit matrix and $\Delta T_3^{(K,u)} = 1/2 - T_3^{(K,u)}$ is the differences between the SM top-type quark and K-th generation VL quark. In the mass eigenstate basis, the left-handed coupling becomes:

$$g_{ZL}^{u,IJ} = \frac{g}{c_W}\left[\left(\frac{1}{2} - Q_u s_W^2\right)\delta^{IJ} - \sum_{K=1,2} \Delta T_3^{(K,u)} V_L^{u*,K+3I} V_L^{u,K+3J}\right]. \tag{B.19}$$

Analogously for the right-handed couplings we obtain:

$$g_{ZR}^{u,IJ} = \frac{g}{c_W}\left[\left(-Q_u s_W^2\right)\delta^{IJ} + \sum_{K=1,2} T_3^{(K,u)} V_R^{u*,K+3I} V_R^{u,K+3J}\right]. \tag{B.20}$$

For bottom-type quark, we obtain the left- and right-handed couplings:

$$g_{ZL}^{d,IJ} = \frac{g}{c_W}\left[\left(-\frac{1}{2} - Q_d s_W^2\right)\delta^{IJ} - \sum_{K=1,2} \Delta T_3^{(K,d)} V_L^{d*,K+3I} V_L^{d,K+3J}\right], \tag{B.21}$$

$$g_{ZR}^{d,IJ} = \frac{g}{c_W}\left[\left(-Q_d s_W^2\right)\delta^{IJ} + \sum_{K=1,2} T_3^{(K,d)} V_R^{d*,K+3I} V_R^{d,K+3J}\right], \tag{B.22}$$

where $\Delta T_3^{(K,d)} = -1/2 - T_3^{(K,d)}$. The left-handed couplings of the Z in $X^{5/3}$ quarks can be written as:

$$\mathcal{L}_Z = \frac{g}{c_W}\left(\bar{X}_{1L}^{5/3}, \bar{X}_{2L}^{5/3}\right)\cdot\left[\begin{pmatrix} T_3^{(1,X)} & \\ & T_3^{(2,X)} \end{pmatrix} - Q_X s_W^2 \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}\right]\gamma^\mu \cdot \begin{pmatrix} X_{1L}^{5/3} \\ X_{2L}^{5/3} \end{pmatrix} Z_\mu, \tag{B.23}$$

where $Q_X = 5/3$. In the mass eigenstate, the coupling becomes:

$$g_{ZL}^{X^{5/3},IJ} = \frac{g}{c_W}\left[-Q_X s_W^2 \delta^{IJ} + \sum_{K=1,2} T_3^{(K,X)} V_L^{X^{5/3}*,KI} V_L^{X^{5/3},KJ}\right]. \tag{B.24}$$

The right-handed couplings are

$$g_{ZR}^{X^{5/3},IJ} = \frac{g}{c_W}\left[-Q_X s_W^2 \delta^{IJ} + \sum_{K=1,2} T_3^{(K,X)} V_R^{X^{5/3}*,KI} V_R^{X^{5/3},KJ}\right]. \tag{B.25}$$

For exotic quark $Y^{-4/3}$ we obtain the left- and right-handed couplings are:

$$g_{ZL}^{Y^{-4/3},IJ} = \frac{g}{c_W}\left[-Q_Y s_W^2 \delta^{IJ} + \sum_{K=1,2} T_3^{(K,Y)} V_L^{Y^{-4/3}*,KI} V_L^{Y^{-4/3},KJ}\right], \tag{B.26}$$

$$g_{ZR}^{Y^{-4/3},IJ} = \frac{g}{c_W}\left[-Q_Y s_W^2 \delta^{IJ} + \sum_{K=1,2} T_3^{(K,Y)} V_R^{Y^{-4/3}*,KI} V_R^{Y^{-4/3},KJ}\right], \tag{B.27}$$

where $Q_Y = -4/3$. Similarly, the left- and right-handed couplings of the exotic quarks $X^{8/3}$ and $Y^{-7/3}$ can be expressed as:

$$g_{ZL}^{X^{8/3}} = g_{ZR}^{X^{8/3}} = \frac{g}{c_W}\left[T_3^{(K,X^{8/3})} - Q_{X^{8/3}} s_W^2\right], \tag{B.28}$$

$$g_{ZL}^{Y^{-7/3}} = g_{ZR}^{Y^{-7/3}} = \frac{g}{c_W}\left[T_3^{(K,Y^{-7/3})} - Q_{Y^{-7/3}} s_W^2\right], \tag{B.29}$$

where $Q_{X^{8/3}} = 8/3$ and $Q_{Y^{-7/3}} = -7/3$.

B.3 Higgs boson couplings

In the interaction basis, the Yukawa interactions in top-type quarks can be written as:

$$\mathcal{L}_H = \frac{1}{v}\left(\bar{u}_L^1, \bar{u}_L^2, \bar{u}_L^3, \bar{U}_{1L}, \bar{U}_{2L}\right) \cdot [M_u - M] \cdot \begin{pmatrix} u_R^1 \\ u_R^2 \\ u_R^3 \\ U_{1R} \\ U_{2R} \end{pmatrix} h + h.c., \tag{B.30}$$

with:

$$M = \begin{pmatrix} 0_{3\times 3} & & \\ & M_1 & \\ & & M_2 \end{pmatrix}. \tag{B.31}$$

In the mass eigenstate basis the coupling of top-type quark reads :

$$C^{u,IJ} = \frac{M_u^{diag,IJ}}{v} - \sum_{K=1,2} \frac{M_K}{v} V_L^{u*,K+3I} V_R^{u,K+3J}. \tag{B.32}$$

For bottom-type quark, we obtain:

$$C^{d,IJ} = \frac{M_d^{diag,IJ}}{v} - \sum_{K=1,2} \frac{M_K}{v} V_L^{d*,K+3I} V_R^{d,K+3J}. \tag{B.33}$$

The Higgs is also allowed to couples to exotic charged VL quarks $X^{5/3}/Y^{-4/3}$ if the scenario contains more than one of them: in this case, one can consider the formulas above

and removing the SM quark part:

$$C^{X,IJ} = \frac{M_X^{diag,IJ}}{v} - \sum_{K=1,2} \frac{M_K}{v} V_L^{X^{5/3}*,KI} V_R^{X^{5/3},KJ} , \tag{B.34}$$

$$C^{Y,IJ} = \frac{M_Y^{diag,IJ}}{v} - \sum_{K=1,2} \frac{M_K}{v} V_L^{Y^{-4/3}*,KI} V_R^{Y^{-4/3},KJ} . \tag{B.35}$$

C Contributions to the S,T parameters from VL quarks

Type of model	$\Pi_x(p^2) =$
A.1.1	$\Pi_x^{(A)}(p^2)$
A.1.2	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2)$
A.1.3	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2) + \Pi_x^{(D)}(p^2)$
A.2.1	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.2.2	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2) + \Pi_x^{(E)}(p^2)$
A.3.1	$\Pi_x^{(A)}(p^2)$
A.3.2	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2)$
A.3.3	$\Pi_x^{(A)}(p^2)$
A.3.4	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.1	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2)$
A.4.2	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.3	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.4	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2)$
A.4.5	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2)$
A.4.6	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.7	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.8	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.9	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.10	$\Pi_x^{(A)}(p^2) + \Pi_x^{(C)}(p^2)$
A.4.11	$\Pi_x^{(A)}(p^2) + \Pi_x^{(B)}(p^2) + \Pi_x^{(C)}(p^2)$

Table 6. The $\Pi_x(p^2)$ $(x = 11, 33, 3Q)$ in the two VL multiplets.

The contributions to the S and T parameters (the oblique corrections) can be written in general form for the contribution of VL particles circulating in the loop for the one-loop two point functions in terms of the couplings of these particles. The generic couplings to W, Z are given explicitly in the previous section of the Appendix. In the VL quark model, the general formulas for S, T and U parameters are given by $\Pi_{11}(p^2)$, $\Pi_{33}(p^2)$, $\Pi_{3Q}(p^2)$ and derivative of

them with respect to p^2. The $\Pi_x(p^2)$ $(x = 11, 33, 3Q)$ can be decomposed into the multiple parts $\Pi_x^{(i)}(p^2)$ $(i = A, B, \cdots, E)$ which are based on internal particles in loop diagrams:

$$\Pi_x(p^2) = \sum_i \Pi_x^{(i)}(p^2)\,. \tag{C.1}$$

The results of $\Pi_x(p^2)$ in all possible models under our assumptions are listed in Table 6.

The contributions of part A of $\Pi_{11}(p^2)$, loops of combinations of up- and bottom-type quarks, are given by:

$$\Pi_{11}^{(A)}(p^2) = \frac{1}{g^2}\sum_I \sum_J \left[\left(|g_{WL}^{IJ}|^2 + |g_{WR}^{IJ}|^2\right)\Pi_T^{LL} + 2\mathrm{Re}\left(g_{WL}^{*,IJ} g_{WR}^{IJ}\right)\Pi_T^{LR}\right](p^2; u_I, d_J)\,, \tag{C.2}$$

where $I, J = 1, 2, \cdots, 5$. We define the two point functions as:

$$\Pi_T^{LL}(p^2; f_1, f_2) = -\frac{N_c}{16\pi^2}\left[(4-2D)B_{22} - 2p^2(B_1 + B_{21})\right](p^2, m_{f_1}, m_{f_2})\,, \tag{C.3}$$

$$\Pi_T^{LR}(p^2; f_1, f_2) = -\frac{N_c}{16\pi^2} 2 m_{f_1} m_{f_2} B_0(p^2, m_{f_1}, m_{f_2})\,, \tag{C.4}$$

where N_c is the color factor and B_i are the Passarino-Veltman functions, which are defined by [61]. They satisfy the following relation at $p^2 = 0$:

$$\Pi_T^{LL}(0; f, f) + \Pi_T^{LR}(0; f, f) = 0\,. \tag{C.5}$$

The part B, loops of combinations of top-type quarks and $X_{L/H}^{5/3}$, can be parametrised as:

$$\begin{aligned}
&\Pi_{11}^{(B)}(p^2) = \\
&\frac{1}{g^2}\sum_I \Bigg\{ \left[\left(|g_{WL}^{X^{5/3},4I}|^2 + |g_{WR}^{X^{5/3},4I}|^2\right)\Pi_T^{LL} + 2\mathrm{Re}\left(g_{WL}^{X^{5/3}*,4I} g_{WR}^{X^{5/3},4I}\right)\Pi_T^{LR}\right](p^2; X_L^{5/3}, u_I) \\
&+ \left[\left(|g_{WL}^{X^{5/3},5I}|^2 + |g_{WR}^{X^{5/3},5I}|^2\right)\Pi_T^{LL} + 2\mathrm{Re}\left(g_{WL}^{X^{5/3}*,5I} g_{WR}^{X^{5/3},5I}\right)\Pi_T^{LR}\right](p^2; X_H^{5/3}, u_I)\Bigg\}\,.
\end{aligned} \tag{C.6}$$

The part C, loops of combinations of bottom-type quarks and $Y_{L/H}^{-4/3}$, contributes to:

$$\begin{aligned}
&\Pi_{11}^{(C)}(p^2) = \\
&\frac{1}{g^2}\sum_I \Bigg\{ \left[\left(|g_{WL}^{Y^{-4/3},I4}|^2 + |g_{WR}^{Y^{-4/3},I4}|^2\right)\Pi_T^{LL} + 2\mathrm{Re}\left(g_{WL}^{Y^{-4/3}*,I4} g_{WR}^{Y^{-4/3},I4}\right)\Pi_T^{LR}\right](p^2; d_I, Y_L^{-4/3}) \\
&+ \left[\left(|g_{WL}^{Y^{-4/3},I5}|^2 + |g_{WR}^{Y^{-4/3},I5}|^2\right)\Pi_T^{LL} + 2\mathrm{Re}\left(g_{WL}^{Y^{-4/3}*,I5} g_{WR}^{Y^{-4/3},I5}\right)\Pi_T^{LR}\right](p^2; d_I, Y_H^{-4/3})\Bigg\}\,.
\end{aligned} \tag{C.7}$$

The part D, loops of combinations of $X_{L/H}^{5/3}$ and $X^{8/3}$, gives:

$$\Pi_{11}^{(D)}(p^2) = \frac{1}{g^2}\sum_{K=1,2}\Bigg\{\left[\left(|g_{WL}^{X^{8/3},K1}|^2+|g_{WR}^{X^{8/3},K1}|^2\right)\Pi_T^{LL}+2\mathrm{Re}\left(g_{WL}^{X^{8/3}*,K1}g_{WR}^{X^{8/3},K1}\right)\Pi_T^{LR}\right](p^2;X_K^{8/3},X_L^{5/3})$$
$$+\left[\left(|g_{WL}^{X^{8/3},K2}|^2+|g_{WR}^{X^{8/3},K2}|^2\right)\Pi_T^{LL}+2\mathrm{Re}\left(g_{WL}^{X^{8/3}*,K2}g_{WR}^{X^{8/3},K2}\right)\Pi_T^{LR}\right](p^2;X_K^{8/3},X_H^{5/3})\Bigg\}. \tag{C.8}$$

The part E, loops of combinations of $Y_{L/H}^{-4/3}$ and $Y^{-7/3}$, evaluates to:

$$\Pi_{11}^{(E)}(p^2) = \frac{1}{g^2}\sum_{K=1,2}\Bigg\{\left[\left(|g_{WL}^{Y^{-7/3},1K}|^2+|g_{WR}^{Y^{-7/3},1K}|^2\right)\Pi_T^{LL}+2\mathrm{Re}\left(g_{WL}^{Y^{-7/3}*,1K}g_{WR}^{Y^{-7/3},1K}\right)\Pi_T^{LR}\right](p^2;Y_L^{-4/3},Y_K^{-7/3})$$
$$+\left[\left(|g_{WL}^{Y^{-7/3},2K}|^2+|g_{WR}^{Y^{-7/3},2K}|^2\right)\Pi_T^{LL}+2\mathrm{Re}\left(g_{WL}^{Y^{-7/3}*,2K}g_{WR}^{Y^{-7/3},2K}\right)\Pi_T^{LR}\right](p^2;Y_H^{-4/3},Y_K^{-7/3})\Bigg\}. \tag{C.9}$$

The part A of $\Pi_{33}(p^2)$, loops of combinations of top-type/bottom-type quarks, is given

by:

$$
\begin{aligned}
&\Pi_{33}^{(A)}(p^2) = \\
&\quad \sum_I \sum_{K=1,2} \Bigg\{ \left[\left(\frac{1}{2} - \Delta T_3^{(K,u)} |V_L^{u,K+3I}|^2 \right)^2 + \left(T_3^{(K,u)} |V_R^{u,K+3I}|^2 \right)^2 \right] \Pi_T^{LL}(p^2; u_I, u_I) \\
&\quad + \left[T_3^{(K,u)} |V_R^{u,K+3I}|^2 - 2\Delta T_3^{(K,u)} T_3^{(K,u)} |V_L^{u,K+3I}|^2 |V_R^{u,K+3I}|^2 \right] \Pi_T^{LR}(p^2; u_I, u_I) \\
&\quad + \left[\left(-\frac{1}{2} - \Delta T_3^{(K,d)} |V_L^{d,K+3I}|^2 \right)^2 + \left(T_3^{(K,d)} |V_R^{d,K+3I}|^2 \right)^2 \right] \Pi_T^{LL}(p^2; d_I, d_I) \\
&\quad - \left[T_3^{(K,d)} |V_R^{d,K+3I}|^2 + 2\Delta T_3^{(K,d)} T_3^{(K,d)} |V_L^{d,K+3I}|^2 |V_R^{d,K+3I}|^2 \right] \Pi_T^{LR}(p^2; d_I, d_I) \Bigg\} \\
&\; + 2 \sum_{I<J} \sum_{K=1,2} \Bigg\{ \\
&\quad \left[\left(\Delta T_3^{(K,u)} \right)^2 |V_L^{u,K+3I}|^2 |V_L^{u,K+3J}|^2 + \left(T_3^{(K,u)} \right)^2 |V_R^{u,K+3I}|^2 |V_R^{u,K+3J}|^2 \right] \Pi_T^{LL}(p^2; u_I, u_J) \\
&\quad -2 \left[\Delta T_3^{(K,u)} T_3^{(K,u)} \mathrm{Re} \left(V_L^{u,K+3I} V_L^{u*,K+3J} V_R^{u*,K+3I} V_R^{u,K+3J} \right) \right] \Pi_T^{LR}(p^2; u_I, u_J) \\
&\quad + \left[\left(\Delta T_3^{(K,d)} \right)^2 |V_L^{d,K+3I}|^2 |V_L^{d,K+3J}|^2 + \left(T_3^{(K,d)} \right)^2 |V_R^{d,K+3I}|^2 |V_R^{d,K+3J}|^2 \right] \Pi_T^{LL}(p^2; d_I, d_J) \\
&\quad -2 \left[\Delta T_3^{(K,d)} T_3^{(K,d)} \mathrm{Re} \left(V_L^{d,K+3I} V_L^{d*,K+3J} V_R^{d*,K+3I} V_R^{d,K+3J} \right) \right] \Pi_T^{LR}(p^2; d_I, d_J) \Bigg\} . \qquad \text{(C.10)}
\end{aligned}
$$

The part B, loops of $X_L^{5/3}$ and $X_H^{5/3}$, is:

$$
\begin{aligned}
\Pi_{33}^{(B)}(p^2) = \sum_{I=1,2} \sum_{J=1,2} \Bigg\{ & T_3^{(I,X)} T_3^{(J,X)} \Bigg[\\
& \left(|V_L^{X^{5/3},I1}|^2 |V_L^{X^{5/3},J1}|^2 + |V_R^{X^{5/3},I1}|^2 |V_R^{X^{5/3},J1}|^2 \right) \Pi_T^{LL}(p^2; X_L^{5/3}, X_L^{5/3}) \\
& +2 |V_L^{X^{5/3},I1}|^2 |V_R^{X^{5/3},J1}|^2 \Pi_T^{LR}(p^2; X_L^{5/3}, X_L^{5/3}) \\
& +2 |V_L^{X^{5/3},I2}|^2 |V_R^{X^{5/3},J2}|^2 \Pi_T^{LR}(p^2; X_H^{5/3}, X_H^{5/3}) \\
& + \left(|V_L^{X^{5/3},I2}|^2 |V_L^{X^{5/3},J2}|^2 + |V_R^{X^{5/3},I2}|^2 |V_R^{X^{5/3},J2}|^2 \right) \Pi_T^{LL}(p^2; X_H^{5/3}, X_H^{5/3}) \\
& +2 \Big(\mathrm{Re} \left(V_L^{X^{5/3}*,I1} V_L^{X^{5/3},I2} V_L^{X^{5/3},J1} V_L^{X^{5/3}*,J2} \right) \\
& +\mathrm{Re} \left(V_R^{X^{5/3}*,I1} V_R^{X^{5/3},I2} V_R^{X^{5/3},J1} V_R^{X^{5/3}*,J2} \right) \Big) \Pi_T^{LL}(p^2; X_L^{5/3}, X_H^{5/3}) \\
& +2 \mathrm{Re} \left(V_L^{X^{5/3}*,I1} V_L^{X^{5/3},I2} V_R^{X^{5/3},J1} V_R^{X^{5/3}*,J2} \right) \Pi_T^{LR}(p^2; X_L^{5/3}, X_H^{5/3}) \Bigg] \Bigg\} . \qquad \text{(C.11)}
\end{aligned}
$$

The part C, loops of $Y_L^{-4/3}$ and $Y_H^{-4/3}$, is:

$$
\begin{aligned}
\Pi_{33}^{(C)}(p^2) = \sum_{I=1,2}\sum_{J=1,2}\Bigg\{ & T_3^{(I,Y)}T_3^{(J,Y)}\Bigg[\\
& \left(|V_L^{Y^{-4/3},I1}|^2|V_L^{Y^{-4/3},J1}|^2+|V_R^{Y^{-4/3},I1}|^2|V_R^{Y^{-4/3},J1}|^2\right)\Pi_T^{LL}(p^2;Y_L^{-4/3},Y_L^{-4/3}) \\
& +2|V_L^{Y^{-4/3},I1}|^2|V_R^{Y^{-4/3},J1}|^2\Pi_T^{LR}(p^2;Y_L^{-4/3},Y_L^{-4/3}) \\
& +2|V_L^{Y^{-4/3},I2}|^2|V_R^{Y^{-4/3},J2}|^2\Pi_T^{LR}(p^2;Y_H^{-4/3},Y_H^{-4/3}) \\
& +\left(|V_L^{Y^{-4/3},I2}|^2|V_L^{Y^{-4/3},J2}|^2+|V_R^{Y^{-4/3},I2}|^2|V_R^{Y^{-4/3},J2}|^2\right)\Pi_T^{LL}(p^2;Y_H^{-4/3},Y_H^{-4/3}) \\
& +2\left(\mathrm{Re}\left(V_L^{Y^{-4/3}*,I1}V_L^{Y^{-4/3},I2}V_L^{Y^{-4/3},J1}V_L^{Y^{-4/3}*,J2}\right)\right. \\
& \left.+\,\mathrm{Re}\left(V_R^{Y^{-4/3}*,I1}V_R^{Y^{-4/3},I2}V_R^{Y^{-4/3},J1}V_R^{Y^{-4/3}*,J2}\right)\right)\Pi_T^{LL}(p^2;Y_L^{-4/3},Y_H^{-4/3}) \\
& +2\mathrm{Re}\left(V_L^{Y^{-4/3}*,I1}V_L^{Y^{-4/3},I2}V_R^{Y^{-4/3},J1}V_R^{Y^{-4/3}*,J2}\right)\Pi_T^{LR}(p^2;Y_L^{-4/3},Y_H^{-4/3})\Bigg]\Bigg\}.
\end{aligned}
\tag{C.12}
$$

The part D/E, loops of $X^{8/3}/Y^{-7/3}$, are:

$$
\Pi_{33}^{(D)}(p^2) = 2\sum_{K=1,2}\left(T_3^{(K,X^{8/3})}\right)^2\left(\Pi_T^{LL}+\Pi_T^{LR}\right)(p^2;X_K^{8/3},X_K^{8/3})\,, \tag{C.13}
$$

$$
\Pi_{33}^{(E)}(p^2) = 2\sum_{K=1,2}\left(T_3^{(K,Y^{-7/3})}\right)^2\left(\Pi_T^{LL}+\Pi_T^{LR}\right)(p^2;Y_K^{-7/3},Y_K^{-7/3})\,, \tag{C.14}
$$

where the Part D and E are exactly cancelled by the reason of $(\Pi_T^{LL}+\Pi_T^{LR})(p^2;m_f,m_f)$ at $p^2=0$:

$$
\Pi_{33}^{(D)}(p^2=0)=0\,,\qquad \Pi_{33}^{(E)}(p^2=0)=0\,. \tag{C.15}
$$

The part A of $\Pi_{3Q}(p^2)$, loops of combinations of top-type/bottom-type quarks, is:

$$
\begin{aligned}
\Pi_{3Q}^{(A)}(p^2) = {} & \frac{1}{2}\sum_I\left\{Q_u\left(\Pi_T^{LL}+\Pi_T^{LR}\right)(p^2;u_I,u_I)-Q_d\left(\Pi_T^{LL}+\Pi_T^{LR}\right)(p^2;d_I,d_I)\right\} \\
& +\sum_I\sum_{K=1,2}\Bigg\{Q_u\left[-\Delta T_3^{(K,u)}|V_L^{u,K+3I}|^2+T_3^{(K,u)}|V_R^{u,K+3I}|^2\right]\Pi_T^{LL}(p^2;u_I,u_I) \\
& +Q_u\left[T_3^{(K,u)}|V_R^{u,K+3I}|^2-\Delta T_3^{(K,u)}|V_L^{u,K+3I}|^2\right]\Pi_T^{LR}(p^2;u_I,u_I) \\
& +Q_d\left[-\Delta T_3^{(K,d)}|V_L^{d,K+3I}|^2+T_3^{(K,d)}|V_R^{d,K+3I}|^2\right]\Pi_T^{LL}(p^2;d_I,d_I) \\
& +Q_d\left[T_3^{(K,d)}|V_R^{d,K+3I}|^2-\Delta T_3^{(K,d)}|V_L^{d,K+3I}|^2\right]\Pi_T^{LR}(p^2;d_I,d_I)\Bigg\}.
\end{aligned}
\tag{C.16}
$$

The part B/C, loops of $X^{3/5}/Y^{-4/3}$, are:

$$\Pi_{3Q}^{(B)}(p^2) =$$
$$Q_X \sum_{K=1,2} \Bigg\{ T_3^{(K,X)} \Bigg[\left(|V_L^{X^{5/3},K1}|^2 + |V_R^{X^{5/3},K1}|^2 \right) \left(\Pi_T^{LL} + \Pi_T^{LR} \right) (p^2; X_L^{5/3}, X_L^{5/3})$$
$$+ \left(|V_L^{X^{5/3},K2}|^2 + |V_R^{X^{5/3},K2}|^2 \right) \left(\Pi_T^{LL} + \Pi_T^{LR} \right) (p^2; X_H^{5/3}, X_H^{5/3}) \Bigg] \Bigg\}, \tag{C.17}$$
$$\Pi_{3Q}^{(C)}(p^2) =$$
$$Q_Y \sum_{K=1,2} \Bigg\{ T_3^{(K,Y)} \Bigg[\left(|V_L^{Y^{-4/3},K1}|^2 + |V_R^{Y^{-4/3},K1}|^2 \right) \left(\Pi_T^{LL} + \Pi_T^{LR} \right) (p^2; Y_L^{-4/3}, Y_L^{-4/3})$$
$$+ \left(|V_L^{Y^{-4/3},K2}|^2 + |V_R^{Y^{-4/3},K2}|^2 \right) \left(\Pi_T^{LL} + \Pi_T^{LR} \right) (p^2; Y_H^{-4/3}, Y_H^{-4/3}) \Bigg] \Bigg\}. \tag{C.18}$$

The part D/E, loops of $X^{8/3}/Y^{-7/3}$, are:

$$\Pi_{3Q}^{(D)}(p^2) = 2Q_{X^{8/3}} \sum_{K=1,2} T_3^{(K,X^{8/3})} \left(\Pi_T^{LL} + \Pi_T^{LR} \right) (p^2; X_K^{8/3}, X_K^{8/3}), \tag{C.19}$$
$$\Pi_{3Q}^{(E)}(p^2) = 2Q_{Y^{-7/3}} \sum_{K=1,2} T_3^{(K,Y^{-7/3})} \left(\Pi_T^{LL} + \Pi_T^{LR} \right) (p^2; Y_K^{-7/3}, Y_K^{-7/3}), \tag{C.20}$$

where the part D and E at $p^2 = 0$ become

$$\Pi_{3Q}^{(D)}(p^2 = 0) = \Pi_{3Q}^{(E)}(p^2 = 0) = 0. \tag{C.21}$$

D Extra numerical results for VL multiplets

We collect in the present appendix extra numerical results which complete those shown in the main text, in particular concerning the limits for the case of the VL quarks coupling to the second SM generation; these are similar in form to those obtained for the coupling to the first SM generation, but bounds vary considerably in some cases. See figure 11.

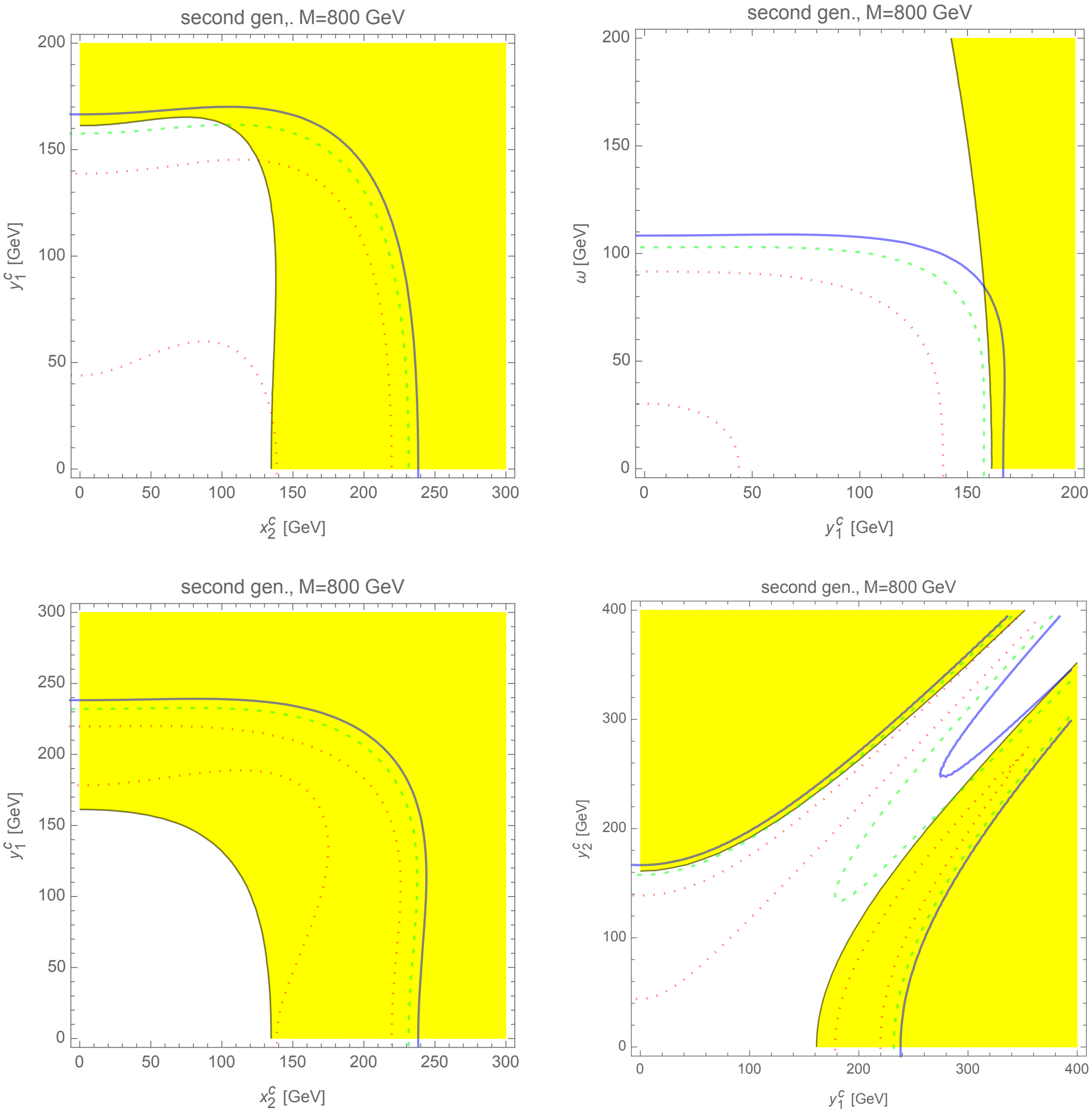

Figure 11. EWP bounds at 1 σ (red-dashed), 2 σ (green-dashed) and 3 σ (blue) for VL quarks coupling with the second SM generations, compared with the region excluded at 3σ by tree-level bounds (yellow region in the left panel). We always chose $M_1 = M_2 = M = 800$ GeV and $\omega = \omega' = 0$ (except for the upper-right plot where $\omega = \omega' \neq 0$). Upper left panel: Singlet $Y = 2/3$ and Doublet $Y = 7/6$. Upper right panel: Doublet $Y = 7/6$ and Triplet $Y = 5/3$. Lower left panel: Singlet $Y = 2/3$ and Doublet $Y = 1/6$. Lower right panel: Doublet $Y = 1/6$ and Doublet $Y = 7/6$. Only the first quadrant is shown as the figures are symmetric with respect to a sign change in the coordinates in the other three quadrants. Note that in the four plots the variables and the units on the axis are not the same, so that they should not be compared directly as they refer to different multiplets.

References

[1] K. Agashe, R. Contino, and A. Pomarol, *The Minimal composite Higgs model*, *Nucl.Phys.* **B719** (2005) 165–187, [hep-ph/0412089].

[2] R. Contino, L. Da Rold, and A. Pomarol, *Light custodians in natural composite Higgs models*, *Phys.Rev.* **D75** (2007) 055014, [hep-ph/0612048].

[3] G. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, *The Strongly-Interacting Light Higgs*, *JHEP* **0706** (2007) 045, [hep-ph/0703164].

[4] O. Matsedonskyi, G. Panico, and A. Wulzer, *Light Top Partners for a Light Composite Higgs*, *JHEP* **1301** (2013) 164, [arXiv:1204.6333].

[5] I. Antoniadis, *A Possible new dimension at a few TeV*, *Phys.Lett.* **B246** (1990) 377–384.

[6] I. Antoniadis, K. Benakli, and M. Quiros, *Finite Higgs mass without supersymmetry*, *New J.Phys.* **3** (2001) 20, [hep-th/0108005].

[7] C. Csaki, C. Grojean, J. Hubisz, Y. Shirman, and J. Terning, *Fermions on an interval: Quark and lepton masses without a Higgs*, *Phys.Rev.* **D70** (2004) 015012, [hep-ph/0310355].

[8] Y. Hosotani, S. Noda, and K. Takenaga, *Dynamical gauge-Higgs unification in the electroweak theory*, *Phys.Lett.* **B607** (2005) 276–285, [hep-ph/0410193].

[9] G. Cacciapaglia, A. Deandrea, and J. Llodra-Perez, *A Dark Matter candidate from Lorentz Invariance in 6D*, *JHEP* **1003** (2010) 083, [arXiv:0907.4993].

[10] N. Arkani-Hamed, A. Cohen, E. Katz, A. Nelson, T. Gregoire, et al., *The Minimal moose for a little Higgs*, *JHEP* **0208** (2002) 021, [hep-ph/0206020].

[11] N. Arkani-Hamed, A. Cohen, E. Katz, and A. Nelson, *The Littlest Higgs*, *JHEP* **0207** (2002) 034, [hep-ph/0206021].

[12] M. Schmaltz and D. Tucker-Smith, *Little Higgs review*, *Ann.Rev.Nucl.Part.Sci.* **55** (2005) 229–270, [hep-ph/0502182].

[13] Y. Hosotani, *Dynamical Mass Generation by Compact Extra Dimensions*, *Phys.Lett.* **B126** (1983) 309.

[14] D. Choudhury, T. M. Tait, and C. Wagner, *Beautiful mirrors and precision electroweak data*, *Phys.Rev.* **D65** (2002) 053002, [hep-ph/0109097].

[15] G. Panico, E. Ponton, J. Santiago, and M. Serone, *Dark Matter and Electroweak Symmetry Breaking in Models with Warped Extra Dimensions*, *Phys.Rev.* **D77** (2008) 115012, [arXiv:0801.1645].

[16] K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, *A Custodial symmetry for Zb anti-b*, *Phys.Lett.* **B641** (2006) 62–66, [hep-ph/0605341].

[17] R. S. Chivukula, R. Foadi, and E. H. Simmons, *Patterns of Custodial Isospin Violation from a Composite Top*, *Phys.Rev.* **D84** (2011) 035026, [arXiv:1105.5437].

[18] CMS B2G twiki page.

[19] ATLAS Exotics twiki page.

[20] F. del Aguila, M. Perez-Victoria, and J. Santiago, *Observable contributions of new exotic quarks to quark mixing*, *JHEP* **0009** (2000) 011, [hep-ph/0007316].

[21] J. Aguilar-Saavedra, *Pair production of heavy $Q = 2/3$ singlets at LHC*, *Phys.Lett.* **B625** (2005) 234–244, [hep-ph/0506187].

[22] C. Anastasiou, E. Furlan, and J. Santiago, *Realistic Composite Higgs Models*, *Phys.Rev.* **D79** (2009) 075003, [arXiv:0901.2117].

[23] J. Aguilar-Saavedra, *Identifying top partners at LHC*, *JHEP* **0911** (2009) 030, [arXiv:0907.3155].

[24] G. Cacciapaglia, A. Deandrea, D. Harada, and Y. Okada, *Bounds and Decays of New Heavy Vector-like Top Partners*, *JHEP* **1011** (2010) 159, [arXiv:1007.2933].

[25] D. Marzocca, M. Serone, and J. Shu, *General Composite Higgs Models*, *JHEP* **1208** (2012) 013, [arXiv:1205.0770].

[26] A. De Simone, O. Matsedonskyi, R. Rattazzi, and A. Wulzer, *A First Top Partner Hunter's Guide*, *JHEP* **1304** (2013) 004, [arXiv:1211.5663].

[27] A. Falkowski, D. M. Straub, and A. Vicente, *Vector-like leptons: Higgs decays and collider phenomenology*, *JHEP* **1405** (2014) 092, [arXiv:1312.5329].

[28] J. Aguilar-Saavedra, *Mixing with vector-like quarks: constraints and expectations*, *EPJ Web Conf.* **60** (2013) 16012, [arXiv:1306.4432].

[29] S. A. Ellis, R. M. Godbole, S. Gopalakrishna, and J. D. Wells, *Survey of vector-like fermion extensions of the Standard Model and their phenomenological implications*, *JHEP* **1409** (2014) 130, [arXiv:1404.4398].

[30] G. Cacciapaglia, A. Deandrea, L. Panizzi, N. Gaur, D. Harada, et al., *Heavy Vector-like Top Partners at the LHC and flavour constraints*, *JHEP* **1203** (2012) 070, [arXiv:1108.6329].

[31] Y. Okada and L. Panizzi, *LHC signatures of vector-like quarks*, *Adv.High Energy Phys.* **2013** (2013) 364936, [arXiv:1207.5607].

[32] M. Buchkremer, G. Cacciapaglia, A. Deandrea, and L. Panizzi, *Model Independent Framework for Searches of Top Partners*, *Nucl.Phys.* **B876** (2013) 376–417, [arXiv:1305.4172].

[33] C. Delaunay, T. Flacke, J. Gonzalez-Fraile, S. J. Lee, G. Panico, et al., *Light Non-degenerate Composite Partners at the LHC*, arXiv:1311.2072.

[34] D. Barducci, A. Belyaev, M. Buchkremer, G. Cacciapaglia, A. Deandrea, et al., *Framework for Model Independent Analyses of Multiple Extra Quark Scenarios*, *JHEP* **1412** (2014) 080, [arXiv:1405.0737].

[35] M. E. Peskin and T. Takeuchi, *A New constraint on a strongly interacting Higgs sector*, *Phys.Rev.Lett.* **65** (1990) 964–967.

[36] M. E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys.Rev.* **D46** (1992) 381–409.

[37] F. del Aguila and M. J. Bowick, *The Possibility of New Fermions With Δ $I = 0$ Mass*, *Nucl.Phys.* **B224** (1983) 107.

[38] J. Uschersohn and E. Elbaz, *SU(2) X U(1) MODEL WITH QUADRUPLETS*, *Nuovo Cim.* **A80** (1984) 341–356.

[39] A. Deandrea, *Atomic parity violation in cesium and implications for new physics*, *Phys.Lett.* **B409** (1997) 277–282, [hep-ph/9705435].

[40] **Particle Data Group** Collaboration, K. Olive et al., *Review of Particle Physics*, *Chin.Phys.* **C38** (2014) 090001.

[41] **ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, SLD Collaboration, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy Flavour Group** Collaboration, S. Schael et al., *Precision electroweak measurements on the Z resonance*, *Phys.Rept.* **427** (2006) 257–454, [hep-ex/0509008].

[42] G. Chiarelli, *Single Top Physics at Hadron Colliders*, *EPJ Web Conf.* **49** (2013) 04004, [arXiv:1302.1773].

[43] **CMS** Collaboration, S. Chatrchyan et al., *Measurement of the single-top-quark t-channel cross section in pp collisions at $\sqrt{s} = 7$ TeV*, *JHEP* **1212** (2012) 035, [arXiv:1209.4533].

[44] **CDF Collaboration** Collaboration, T. A. Aaltonen et al., *Measurement of the Single Top Quark Production Cross Section and —Vtb— in Events with One Charged Lepton, Large Missing Transverse Energy, and Jets at CDF*, arXiv:1407.4031.

[45] **CMS Collaboration** Collaboration, V. Khachatryan et al., *Measurement of the t-channel single-top-quark production cross section and of the $\mid V_{tb} \mid$ CKM matrix element in pp collisions at $\sqrt{s}$= 8 TeV*, *JHEP* **1406** (2014) 090, [arXiv:1403.7366].

[46] **ATLAS Collaboration** Collaboration, C. Friedrich, *Single top-quark productioncross-section at the LHC in ATLAS*, *EPJ Web Conf.* **71** (2014) 00047.

[47] **ATLAS Collaborations, CMS Collaborations** Collaboration, L. Lista and o. b. o. t. ATLAS, *Top quark production at ATLAS and CMS*, arXiv:1405.5126.

[48] **ATLAS Collaboration, CDF Collaboration, CMS Collaboration, D0 Collaboration** Collaboration, C.-S. Moon, *Production of single top quark - results from the Tevatron and the LHC*, arXiv:1410.3045.

[49] L. Lavoura and J. P. Silva, *The Oblique corrections from vector - like singlet and doublet quarks*, *Phys.Rev.* **D47** (1993) 2046–2057.

[50] M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Ludwig, et al., *Updated Status of the Global Electroweak Fit and Constraints on New Physics*, *Eur.Phys.J.* **C72** (2012) 2003, [arXiv:1107.0975].

[51] M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Kennedy, et al., *The Electroweak Fit of the Standard Model after the Discovery of a New Boson at the LHC*, *Eur.Phys.J.* **C72** (2012) 2205, [arXiv:1209.2716].

[52] **CMS** Collaboration, "Search for vector-like quarks in final states with a single lepton and jets in pp collisions at sqrt s = 8 tev." CMS-B2G-12-017.

[53] G. Brooijmans, R. Contino, B. Fuks, F. Moortgat, P. Richardson, et al., *Les Houches 2013: Physics at TeV Colliders: New Physics Working Group Report*, arXiv:1405.1617.

[54] S. Beauceron, G. Cacciapaglia, A. Deandrea, and J. D. Ruiz-Alvarez, *Fully hadronic decays of a singly produced vectorlike top partner at the LHC*, *Phys.Rev.* **D90** (2014), no. 11 115008, [arXiv:1401.5979].

[55] L. Basso and J. Andrea, *Discovery potential for $T' \to tZ$ in the trilepton channel at the LHC*, arXiv:1411.7587.

[56] R. Barcelo, A. Carmona, M. Chala, M. Masip, and J. Santiago, *Single Vectorlike Quark Production at the LHC*, *Nucl.Phys.* **B857** (2012) 172–184, [arXiv:1110.5914].

[57] **ATLAS** Collaboration, *Search for Single Production of Vector-like Quarks Coupling to Light Generations in* 4.64 ifb *of Data at* $sqrts = 7$ *TeV*, .

[58] **ATLAS** Collaboration, *Search for pair and single production of new heavy quarks that decay to a Z boson and a third generation quark in pp collisions at* $\sqrt{s} = 8$ *TeV with the ATLAS detector*, .

[59] D. Barducci, A. Belyaev, M. Buchkremer, J. Marrouche, S. Moretti, et al., *XQCAT: eXtra Quark Combined Analysis Tool*, arXiv:1409.3116.

[60] https://launchpad.net/xqcat.

[61] G. Passarino and M. Veltman, *One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model*, *Nucl.Phys.* **B160** (1979) 151.